Reduced State Feedback Gain Computation

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Mr. William Adams, Jr.

NASA Langley Research Center

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Submitted by

Howard Kaufman

Associate Professor

Electrical and Systems Engineering Department

Rensselaer Polytechnic Institute

Troy, New York 12181

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Abstract

Under NASA Grant NSG 1188, computer programs were developed for determining constant output feedback gains for linear systems subject to both process and measurement uncertainties. However, in order to evaluate the effectiveness of these programs additional study was required of their applicability to the design of controllers for representative aircraft models.

To this effect a sixth order linear longitudinal model and a 17th order linear longitudinal model with five bending modes were used for the design of reduced state controllers for gust alleviation.

Results show that the developed non-gradient based Zangwill-Powell optimization program could indeed be used to design satisfactory output feedback controllers taking into account the needs for reducing vertical accelerations and structural loading effects.

1. INTRODUCTION

Because application of conventional optimal linear regulator theory to flight control systems requires the capability for measuring and feeding back the entire state vector, it is of interest to consider procedures for computing controls which are restricted to be linear feedback functions of a lower dimensional output vector. Such a procedure, however, has its limitations in that the feedback gains will be functions of the initial state vector. In addition, the presence of measurement noise and process uncertainty can lead to additional problems relating to both modelling and computation.

In order that such reduced state feedback control laws might be considered for the design of flight control systems, an extensive study effort was conducted between 1 June 1975 and 30 Nov. 1976 under NASA Grant NSG 1188 monitored by Mr. Ray Hood of the Langley Research Center.(1)

The objectives of this grant were to:

- Develop procedures for computing optimal constant output feedback gains for linear flight control systems taking into account the presence of turbulence, pilot commands, parameter uncertainty, and measurement noise.
- Implement, on NASA Langley's CDC6600, computer programs capable of determining such optimal output feedback gains.
- Compare the performance of the various computational algorithms and investigate various procedures for modelling the system uncertainties.
- Document the program operation for public dissemination.

Toward these goals, working programs were developed for finding output feedback gains for linear systems in order to minimize both infinite and finite time performance indices.[2] Computational procedures included algorithms which require direct computation of the gradient of the index with respect to the gains and the algorithm proposed by Powell[3] and modified by Zangwill[4] which does not require gradient computation. Two different procedures for modelling plant parameter uncertainties were considered.

Significant among the conclusions resulting from these studies were the following:[1]

- Use of a finite time performance index can result in a set of gains which do not stabilize the closed loop system.
- If it is possible at all to stabilize the system with the specified feedback configuration, then the optimization of an infinite time performance index will yield a set of gains that do indeed stabilize the closed loop system.
- Optimization of infinite time indices is less time consuming than the optimization of finite time indices because of the need to solve algebraic and not differential equations. However, the computation procedures for infinite time indices do require initialization with a gain matrix that stabilizes the closed loop system.
- Application of gradient based algorithms to the optimization of infinite time indices can result in divergence. This in particular results when the gradient is computed using the algebraic steady state Riccati solutions to the matrix covariance and co-state equations. These solutions are only valid provided that the gain matrix under consideration stabilizes the closed loop system. Consequently, if during the search process a perturbation results in a destabilizing gain, then the corresponding computed gradient will be meaningless.

Although these previous efforts resulted in a set of computer programs which can be used for finding a set of gains for a given reduced state feedback control configuration, it was necessary that further explorations be made of their utility to flight control system design. In particular the effects of modelling, sensitivity, and stabilization needed consideration with respect to more realistic aircraft models.

Thus under the present grant, NSG 1384, further studies have been made towards the application of the reduced state stochastic infinite time optimization programs to the design of control systems for representative flexible aircraft. Towards this goal the following tasks have been performed:

- Incorporation of a procedure which computes if possible an initial gain matrix.
- Further comparison of both gradient and non-gradient based procedures for designing reduced state feedback flight control systems.
- Evaluation of the reduced state feedback control computation package in designing a gust alleviation controller for a representative flexible aircraft.

Results show that the non-gradient based reduced state feedback control design program can indeed be used for designing acceptable controllers for a 17th order flexible aircraft model. This conclusion was based upon the performance of controllers designed for the reduction of vertical acceleration and structural loads in the presence of a vertical wind gust.

2. Problem Statement

2.1 Process Model

The optimization algorithms discussed in ref. 1 are applicable to systems described by the following set of state equations:

Process: $\dot{X}_p = A_p X_p + B_p U_p + G_p X_n + w_p$ (1)

Reference: $\dot{X}_r = A_r X_r + B_r w_r$ (2)

Disturbance: $\dot{X}_n = A_n X_n + B_n w_n$ (3)

Where: X_p = (NXP x 1) plant state vector

X_r = (NXR x 1) reference state

X_n = (NXN x 1) disturbance state

U_p = (NUP x 1) control vector

w_p = (NXP x 1) process white noise vector with covariance W_p

w_r = (NWR x 1) reference white noise input with covariance W_R

w_n = (NWN x 1) disturbance white noise input with covariance W_n

Given the system defined by equations 1, 2, and 3, the available outputs are to be designated as:

$$Y_p = C_{pp} X_p + C_{pn} X_n + \gamma \quad (4)$$

$$Y_r = C_{rr} X_r \quad (5)$$

$$Y_n = C_{nn} X_n \quad (6)$$

where Y_p = (NYP x 1) process measurement

Y_r = (NYP x 1) reference measurement

Y_n = (NYN x 1) disturbance measurement

and γ = a zero mean white noise disturbance with covariance Γ.

The control U_p is to be of the form:

$$U_p = K_{y_p} Y_p + K_{y_r} Y_r + K_{y_n} Y_n \tag{7}$$

where the gain matrices K_{y_r}, K_{y_r}, K_{y_n} are to be computed so as to minimize:

$$J = \lim_{T\to\infty} \mathcal{E}\left[\frac{1}{T} \int_0^T \left\{(Z_p - Z_r)^T Q(Z_p - Z_r) + U_p^T R U_p\right\} dt\right] \tag{8}$$

and where $\mathcal{E}[\cdot]$ denotes statistical expectation and the controlled variables Z_p and Z_r are of the form: $Z_p = D_{pp} X_p + D_{pn} X_n$; $Z_r = D_{rr} X_r$

2.2 Optimization Procedures

For notational convenience, eqs. 1, 2, 3 will be compressed into the form:

$$\dot{X} = A X + B U_p + V_1 (t) \tag{9}$$

$$\text{where } X = \begin{pmatrix} X_p \\ X_r \\ X_n \end{pmatrix}$$

$$V_1(t) = \begin{pmatrix} W_p + V(t) \\ B_r W_r \\ B_n W_n \end{pmatrix} \qquad \mathcal{E}(V_1 V_1^T) = V$$

$$A = \begin{pmatrix} A_p & 0 & G_p \\ 0 & A_r & 0 \\ 0 & 0 & A_n \end{pmatrix} \qquad B = \begin{pmatrix} B_p \\ 0 \\ 0 \end{pmatrix}$$

Similarly, eqs. 4, 5, 6 can be expressed as:

$$Y = C X + n \tag{10a}$$

and the controlled variables as:

$$Z = DX \tag{10b}$$

where $Y = \begin{pmatrix} y_p \\ y_r \\ y_n \end{pmatrix}$ $\quad n = \begin{pmatrix} \gamma \\ 0 \\ 0 \end{pmatrix}$ $\quad C = \begin{pmatrix} C_{pp} & 0 & C_{pn} \\ 0 & C_{rr} & 0 \\ 0 & 0 & C_{nn} \end{pmatrix}$ $\quad D = \begin{pmatrix} D_{pp} & 0 & D_{pn} \\ 0 & D_{rr} & 0 \end{pmatrix}$

$Z^T = (Z_p^T, Z_r^T)$

Therefore the control U_p can be rewritten as:

$$U_p = KY \tag{11}$$

where $K = [K_{y_p}, K_{y_r}, K_{y_n}]$

With this notation, eq. 8 for the performance index can be rewritten as:

$$J = \lim_{T\to\infty} \mathcal{E}\left[\frac{1}{T} \int_0^T [Z^T Q_a Z + U_p^T R U_p]\, dt\right]$$

where $Q_a = \begin{bmatrix} Q & -Q & 0 \\ -Q & Q & 0 \\ 0 & 0 & 0 \end{bmatrix}$

or

$$J = \lim_{T\to\infty} \mathcal{E}\left[\frac{1}{T} \int_0^T [X^T D^T Q_a D X + X^T C^T K^T R K C X + 2 X^T C^T K^T R n + n^T K^T R K n]\, dt\right]$$

Note that some simplification is possible since

$$\mathcal{E}(X^T C^T K^T R n) = 0$$

Thus, the actual index selected for minimization was:

$$J = \lim_{T\to\infty} \mathcal{E} \frac{1}{T} \int_0^T X^T \; (D^T Q_a D + C^T K^T R \; K \; C) \; X \; dt$$

$$+ \; \mathcal{E}(n^T K^T \; R \; K \; n)$$

or[8,9] $$J = \text{Trace } [(D^T \; Q_a \; D + C^T \; K^T \; R \; K \; C) \; P_S] + \text{Trace } (K^T \; R \; K \; N) \tag{12}$$

where N = covariance of n

and $$P_S = \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathcal{E}(X \; X^T) \; dt$$

Note that the last term in eq. 12 is a penalty on the gains. Increasing R and/or N therefore has the effect of reducing the magnitude of the resulting gains. If this integral does not converge, then it is common to use the steady state value[5,6].

$$P_S = \lim_{T\to\infty} \quad (X(t) \; X^T(t)).$$

It is of importance to note that provided (A-B K C) is a stable matrix, P_s may be found by solving the linear matrix equation:[8]

$$0 = (A\text{-}B \; K \; C) \; P_s + P_s \; (A\text{-}B \; K \; C)^T + B \; K \; N \; K^T \; B^T + V \tag{13}$$

Thus for a given set of gains it is possible to compute the performance index from (13) if the steady state covariance matrix P_S is available.

3. Computational Procedures

3.1 Non-gradient based Optimization

Since the performance index (eq. 12) is easily evaluated given a value for the gain matrix K, the Zangwill-Powell(3,4) method which does not require gradient computation is very attractive for optimization. In particular, the IMSL sub-routine ZXPOWL was used for implementation.(10)

Letting NK denote the number of unknown gains to be determined, each iteration of the procedure begins with NK single dimension minimizations along NK linearly independent directions ξ_1, ξ_2, ..., ξ_{NK} each starting from K_o, the most recently available gain matrix. Initially K_o is user selected and the directions correspond to the coordinate vectors so that during the first set of minimizations only one gain element is changed at a time. Subsequent iterations define the direction vector set (ξ_2, ξ_3, ..., ξ_{NK}, ξ), where ξ is chosen such if the objective were quadratic, after k iterations, the last k of the direction vectors would be mutually conjugate. These revised directions are then used for the next iteration.(3,4)

This procedure is especially useful for the index defined by eqs. 12, 13. In this case if the algebraic solution of the steady state version of eq. 13 is to be meaningful, then the gain K must stabilize the closed loop system (A-B K C). If K is not stabilizing, then $J = \infty$. If during the process of searching along a particular direction vector ξ_L, perturbations are such that (A-B K C) becomes unstable, then under program control, the size of the perturbation will be reduced (to zero if necessary) prior to the subsequent search along the next direction vector. This is in contrast with conventional gradient type search procedures which do not have other search direction vectors available when instability results.(5)

To account for the above stability problem using the program ZXPOWL, the eigenvalues ξ_i of (A-B K C) were computed for each perturbed value of K, and J was then set equal to 10^{50} whenever an eigenvalue was found to be greater than or equal to zero.

To insure convergence of the procedure, it is necessary that the initial gain K^o stabilize the closed loop matrix, $(A-B\ K^oC)$. Such a matrix, if one exists at all for the permissible feedback structure, may be found by defining an initial phase to the procedure in which the performance index to be minimized with respect to K^o is

$$J_1 = \text{Maximum real part of the eigenvalues of } (A-B\ K^oC).$$

As soon as a gain vector is found which is such that all eigenvalues of $(A-B\ K^oC)$ have negative real parts, then the final phase can be undertaken in which the index is that defined by eqs. 12, 13.

3.2 Gradient based computation procedures

In order to compare the performance of the non-gradient based Zangwill-Powell procedures with gradient based algorithms, a program implementing the sequential unconstrained minimization technique (SUMT)[11] was also considered.

This program is modular in structure so as to facilitate changes, and uses a series of control, special purpose, and user supplied subroutines in order to solve the general mathematical programming problem:

Determine the vector $\underline{X}$ so as to

(a) minimize $F(\underline{X})$

subject to:

$$R_j(\underline{X}) \geq 0 \qquad j = 1, \ldots, M$$

$$R_j(\underline{X}) = 0 \qquad j = M + 1, \ldots, (M + MZ)$$

This is done by solving a sequence of unconstrained problems whose solutions approach the solution of (a).

Specifically, the SUMT procedure uses the function defined as:

$$P(\underline{X},r) = F(\underline{X}) - r \sum_{j=1}^{M} \ln(R_j(\underline{X})) + \frac{1}{r} \sum_{j=M+1}^{M+MZ} (R_j(\underline{X}))^2$$

Using a designated search procedure, a sequence of $P(\underline{X},r_i)$ is minimized for $r = r_1, r_2, \ldots, r_k$, where $r_{i+1} = r_{i/c}$ and $c > 1$. Under suitable conditions, the minima of P represented by $\underline{X}(r_1), \underline{X}(r_2), \ldots, \underline{X}(r_k)$ exist and approach a solution of the constrained problem (a) as $r_k \to 0$.(1) That is:

$$\lim_{r_k \to 0} \underline{X}(r_k)) = \underline{X}^*$$

$$\lim_{r_k \to 0} F(\underline{X}(r_k)) = F(\underline{X}^*)$$

Note that the equality constraints are satisfied only in the limit as $r_k \to 0$. An extrapolation option is available which can, in some cases, accelerate the convergence. To start with, initial values $\underline{X}_o$ for $\underline{X}$ and r_1 for r, must be available. These can either both be read into the program, or r_1 can be computed using one of two options which have been programmed; namely:

OPTION 1: Find r to minimize

$$\nabla_x^T P(\underline{X}_o, r) [\nabla_{xx} P(\underline{X}_o, r)]^{-1} \nabla_x P(\underline{X}_o, r)$$

This is useful if at least one

$$R_j(\underline{X}_o) \approx 0$$

OPTION 2: Ignore the equality constraints and minimize

$$||\nabla_x P(\underline{X}_o, r)||^2$$

Furthermore, if $\underline{X}_o$ results in one or more of the inequality constraints not being satisfied, then the program operates in a feasibility phase by forming an auxiliary objective function equal to the negative of the sum of all the violated inequality constraints. When a constraint is noted to become feasible during the minimization of the auxiliary objective, it is removed and included in the effective constraint set.

To apply the SUMT program to the design of a stochastic reduced state feedback controller, the vector of unknowns $\underline{x}$ must be associated with the elements in the gain matrix and the constraints R_j must be formulated so as to yield a stable closed loop system. In particular, the equality constraints were used, and R_1 was formulated to be the negative of the maximum real part over all eigenvalues of the closed loop system matrix.

4. Results and Discussions

Although most of the contractual effort was expended on the application of optimal reduced state feedback gains to realistic aircraft models, some preliminary activities were devoted to finding procedures for computing an initial stabilizing gain matrix and to comparing the gradient based SUMT procedures with the Zangwill-Powell approach.

4.1 Gain initialization

Since use of the Zangwill-Powell method in conjunction with eqs. 12,13 requires that the initial gain matrix stabilize the closed loop matrix (A-B K C), it is important that a procedure be incorporated for finding such a gain matrix, if it exists, for a specified feedback structure.

After studying several possibilities,[12] it was decided to use the Zangwill-Powell procedure to minimize, with respect the gains, the maximum real part of the closed loop eigenvalues. This procedure has been incorporated into the program, and is called,if the original specified gain yields an eigenvalue with a positive real part. If after a specified number of iterations (ITMAX), the Zangwill-Powell procedure does not find a stabilizing gain matrix, then a message is printed, and the program stops.

4.2 Gradient based optimization procedures

In order to compare the performance of the non-gradient based Zangwill-Powell procedures with gradient based algorithms, a program implementing the sequential unconstrained minimization technique (SUMT) discussed in Section 3.2 was developed and applied to both third and sixth order linearized longitudinal models.[13] In general, it was observed that the SUMT procedure required more computer time than the Zangwill-Powell method to converge; in fact, in many cases the final SUMT performance index exceeded that reached by the Z-P procedure.[13] These relative inefficiencies were attributed not only to the need for gradient computation, but also to the requirement for optimizing a sequence of many unconstrained problems.

4.3 Application to representative aircraft models

In order to evaluate the effectiveness of the infinite time reduced state feedback controller program, various controller configurations were developed according to NASA suggestions on two analytical models of the TIFS aircraft. The TIFS is a fly by wire C-131 aircraft owned by CALSPAN.[14] With its onboard computer and separate controller for all six rigid body degrees of freedom, it is a unique facility for control system and handling qualities research.

4.3.1 Gust alleviation using a sixth order longitudinal model

4.3.1.1 Flight control problem definition

Initially a modified six-dimensional version of the TIFS aircraft perturbed by a vertical wind gust was used for evaluation.[15] The corresponding variable definitions in accordance with eqs. 1-6 were as follows:

Plant state:

$$\underset{\sim}{x}_p = \begin{pmatrix} q \\ \Delta\theta \\ \Delta V \\ \Delta\alpha \\ \delta e \\ \delta z \end{pmatrix} = \begin{pmatrix} \text{pitch rate} \\ \text{pitch angle} \\ \text{longitudinal velocity} \\ \text{angle of attack} \\ \text{elevator deflection} \\ \text{direct lift flap deflection} \end{pmatrix}$$

Disturbance

$$x_n = (\alpha_g) = (\text{gust induced attack angle})$$

Plant control:

$$\underset{\sim}{u}_p = \begin{pmatrix} \delta_{ec} \\ \delta_{zc} \end{pmatrix} \qquad \begin{pmatrix} \text{elevator command} \\ \text{lift flap command} \end{pmatrix}$$

Sensed outputs:

$$y = \begin{pmatrix} q \\ \Delta\theta \\ \alpha \\ \alpha_g \\ \alpha-\alpha_g \end{pmatrix} \qquad \begin{pmatrix} \text{pitch rate} \\ \text{pitch angle} \\ \text{angle of attack} \\ \text{gust attack angle} \\ \text{composite angle of attack} \end{pmatrix}$$

Controlled outputs:

$$\underline{z} = \begin{pmatrix} n_{z1} \\ n_{z2} \end{pmatrix} = \begin{pmatrix} \text{vertical acceleration at point } \# 1 \\ \text{vertical acceleration at point } \# 2 \end{pmatrix}$$

The structural matrices (eqs. 1-6) used for design corresponded to the climb condition, i.e., h = 1524 m, V = 106 m/s, and are (units in radians and ft/sec.)

$$A_p = \begin{array}{c} \begin{array}{cccccc} q & \Delta\theta & \Delta V & \Delta\alpha & \delta_e & \delta_c \end{array} \\ \begin{pmatrix} -.1686 & .000035 & .000231 & -.486 & -4.3773 & -.19948 \\ 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & -32.17 & -.0143 & 18.027 & 0. & -3.0933 \\ 1. & 0.000013 & -.000531 & -1.223 & -.1273 & -.2667 \\ 0. & 0. & 0. & 0. & -20. & 0. \\ 0. & 0. & 0. & 0. & 0. & -40. \end{pmatrix} \end{array}$$

$$B_p = \begin{pmatrix} 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 20. & 0. \\ 0. & 40. \end{pmatrix} \qquad G_p = \begin{pmatrix} -.486 \\ 0 \\ 18.027 \\ -1.223 \\ 0. \\ 0. \end{pmatrix}$$

$$An = -.2784 \quad , \quad Bn = .01815$$

$$C_{pp} = \begin{pmatrix} 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. \end{pmatrix}$$

$$C_{pn} = (0,0,0,1,-1)^T$$

$$D_{pp} = \begin{pmatrix} 64.63 & .00318 & .176 & 444.2 & 212.1 & 100.4 \\ -61.82 & .00580 & .193 & 407.8 & -116.2 & 85.5 \end{pmatrix}$$

$$D_{pn} = \begin{pmatrix} 444.2 \\ 407.8 \end{pmatrix}$$

Sensor noise standard deviations:

$$\sigma_q = .5 \text{ deg/sec}$$
$$\sigma_\theta = .2 \text{ deg.}$$
$$\sigma_\alpha = \sigma_{\alpha g} = .2 \text{ deg.}$$

For design purposes the gust was selected to correspond with a standard deviation vertical wind speed of 15 feet/second. However because of a programming error (discussed in more detail in 4.3.2) the computed gust standard deviation was in reality $15/\sqrt{\pi}$, or 8.46 ft/sec = 2.58 m/s. Thus the gust would correspond to a situation encountered somewhere between normal and cumulus type sky conditions.

The performance index weights were then chosen to be approximately equal to the inverse square of the maximum permissible values of the associated variables. This resulted in

$$J = \lim_{T \to \infty} \frac{1}{T} \; E \int_0^T \left(\frac{n_{31}^2}{16^2} + \frac{n_{32}^2}{16^2} + \frac{\delta_{ec}^2}{.4^2} + \frac{\delta_{zc}^2}{.6^2} \right) dt \qquad (14)$$

4.3.1.2 Simulation Results

Initial studies concerned with the effects of various feedback signals were followed up with a study of the influence of sensor noise and different initial gains. Then using a typical feedback configuration, the sensitivity to flight condition changes and gust variance was also investigated. In each case evaluations were made relative to the resulting value of the performance index and the reduction in root mean square vertical acceleration, measured over five seconds; comparisons were made with respect to the open loop values:*

$$n_{z1} = 9.75 \text{ ft/sec.}^2$$

$$n_{z2} = 9.28 \text{ ft/sec.}^2$$

Effect of feedback configuration

The following combination of sensed outputs were examined with regard to their effectiveness in gust alleviation:

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* The same pseudo white noise sequence w_n was used in all evaluations.

q, θ, α

q, θ, $(\alpha-\alpha g)$

q, θ, α, αg

q, θ, $(\alpha-\alpha g)$, αg

Examination of the results shown in Table 1 corresponding to an open loop or zero gain initialization show that all configurations reduced the rms vertical acceleration better than 60% and that use of the gust induced angle of attack αg in the feedback (either directly or implicitly through $\alpha-\alpha g$) further improved this effectiveness by an additional 20-40%. Note also that in the presence of sensor noise with the intensities cited in Section 4.3.1, the rms values of the vertical acceleration became moderately large. This motivated an accounting of the sensor noise characteristics in the optimization formulation.

Effects of sensor noise

Based upon the above results, the sensor noise was accounted for by assigning the appropriate variances to the noise term γ of eq. 4. Related results shown in Table 2 show that taking into account the sensor noise in the design phase results in the presence of distinct short period and phugoid modes along with smaller gains and a tremendous reduction (with respect to Table 1) in the vertical acceleration responses. Note, however, that in order to achieve any significant improvement over the open loop case, it becomes necessary to include either a direct or implicit feedback of the gust effect αg.

Sensitivity Studies

In order to evaluate the sensitivity of the design procedures to variations in the initialization and in the model, various perturbations were made to the scenarios of the preceding two cases. First to assess the effect of different initial gains, a gain K^o was found (as discussed in section 4.1 to minimize the maximum real part of the eigenvalues of $(A-BK^oC)$. This gave

$$K^o = \begin{pmatrix} -.001616 & -.09349 & -.002438 \\ -.0000851 & -.002114 & .0003967 \end{pmatrix}$$

with eigenvalues:

-20.01

$-40.$

$-1.345 \pm j\ .7702$

$-\ .1090 \pm j\ .00412$

$-\ .2784$

Using this gain to initialize the optimization in the presence of sensor noise led to results almost identical with those presented in Table 2. However, without sensor noise, the optimal feedback gain between δ_z and q was in all cases greater than 1500 (in magnitude).

To test the sensitivity of the gains with respect to flight condition changes, the Table 1 gains for the (q, θ, α-αg) feedback configuration which was derived for the climb condition was used in simulations at landing (h=61m, V=68 m/s)and cruise (h-3048m, V=150m/s). This resulted in the following rms errors in vertical acceleration:

	n_{31}	n_{32}
Landing	6.20	4.92
Cruise	2.85	2.80

It was noted that although these values were significantly lower than the corresponding open loop values:

	n_{31}	n_{32}
Landing	17.05	12.95
Cruise	12.73	12.25.

they were higher than the corresponding Table 1 values for the climb conditions.

Thus, the gains for the (q, θ, α-α_g) configurations were redesigned for the climb condition, but with a process noise term included to account for the uncertainty. To reflect the expected degree of uncertainty, the covariance of this fictitious process noise was chosen as:

$$\text{DIAG}\ (.00006,\ 0.,\ .00003,\ .00003,\ 0.,\ 0.)$$

This resulted in a set of smaller gains which gave the following rms vertical accelerations:

	n_{31}	n_{32}
Landing	5.31	3.27
Climb	2.20	2.20
Cruise	3.02	3.05

Note that the inclusion of the process uncertainty improved the response at the landing configuration at the expense of small degradations in the response at the other two flight conditions.

Finally, in order to determine the effect of the assigned gust variance on the performance, gains were redesigned for the (q, θ, α-α_g) feedback configuration with the assigned gust variance increased by a factor of four (rms value = 30 ft/sec). Although the gains were approximately doubled in value, it was determined through simulation that both the larger gust gains and the smaller gust gains improved the open loop response by about a factor of three regardless of which gust was being applied.

4.3.1.3 Discussion

On the basis of the above results for the sixth order system, it was concluded that stochastic reduced state feedback design procedures would be useful in the design of aircraft gust alleviation control systems. Thus consideration was subsequently given to high order models which take into account bending modes, additional control surface deflections, and loading effects.

4.3.2 Reduced State Feedback Control of a flexible aircraft

4.3.2.1 Problem Definition

To more realistically evaluate the use of the reduced state feedback control program, consideration was directed towards a 17th order model of the TIFS aircraft which incorporated five bending modes and three first order actuators.[14] This model for the unaugmented TIFS was derived by CALSPAN using a quasistatic reduction on the equations which had been obtained with the FLEXSTAB estimation program.[14]

This data taken from Appendix B of ref. 14 (and reproduced in Appendix B of this report) for each of the 3 flight conditions (cruise, climb, land) was arranged into the format:

state eqn: $$\dot{X}_p = A_p X_p + B_p U_p + G_p X_n \qquad (15)$$

gust eqn: $$\dot{X}_n = A_n X_n + B_n W_n \qquad (16)$$

observation: $$Y_p = C_{pp} X_p + C_{pn} X_n \qquad (17)$$

control: $$U_p = K Y_p \qquad (18)$$

X_p is the aircraft state vector in the body axis system with components:

u	x-velocity (m/s)
w	z-velocity (m/s)
q	pitch rate (r/s)
θ	pitch (r)
n_1	1st bending mode
$\dot{n}_1$	
n_2	2nd bending mode
$\dot{n}_2$	
n_3	3rd bending mode
$\dot{n}_3$	
n_4	4th bending mode
$\dot{n}_4$	
n_5	5th bending mode
$\dot{n}_5$	
δ_{sa}	symmetric aileron (deg)
δ_z	direct lift flap (deg)
δ_e	elevator (deg)

Note that the control surface deflections (all in deg) are treated as states from the actuator dynamics which are forced by the computed command $U_p = (\delta_{sa_c}, \delta_{z_c}, \delta_{e_c})$.

The observation vector y consists of the components:

n_{zp}	vertical acceleration (VA) at pilot station (g)
n_{zcg}	VA at cg (g)
n_{zt}	VA at tail (g)
n_{zwt}	VA at wing tip (g)
n_{zFSF}	VA on side force surface (g)
n_{zast}	VA at point on tail (g)
n_{zRHT}	VA at right horizontal tail (g)
q_{cg}	pitch rate at cg (deg/s)
α_v	Vane angle of attack (deg)
S_R	Root shear (n)
BM_R	Root Bending Moment (n-m)
T_R	Root Torque (n-m)

The vertical wind gust x_n, was generated according to the following equation received from NASA:

$$\dot{X}_n = -\frac{2V_o}{L} X_n + 2\sigma \sqrt{\frac{V_o}{\pi L}}\, \xi(t)$$

where V_o = trim velocity

L = effective length

and $\xi(t)$ is a zero mean, unit variance white noise process

In actuality since the steady state variance of X_n resulting from this equation is $\frac{\sigma^2}{\pi}$, selection of parameters so as to yield a specified σ^2 will really result in a variance lower by a factor of π. Consequently in the ensuing results which were derived for σ = 1m/s, the true gust variance was actually .3183 m^2/s^2. Thus in order to extrapolate the normalized results to typical sky conditions, the following multiplication factors should be used:

Sky condition	Factor
Normal ($\sigma_{X_n} \approx 6$ ft/s)	3
Cumulus ($\sigma_{X_n} \approx 15$ ft/s)	8
Thunderstorm ($\sigma_{X_n} \approx 30$ ft/s)	16

From conversations with personnel at NASA Langley, it was determined that it would be of interest to determine reduced state feedback control gains for minimizing vertical acceleration and wing root bending moment which result from a vertical wind gust. To this effect the following two performance indices were considered:

$$J_1 = \lim_{T\to\infty} \frac{1}{T} \mathcal{E} \int_0^T \left[10^6\ (BMR)^2 + 10^{-6}\ [4\delta_{sa_c}{}^2 + 6\delta_{zc}{}^2 + 25\delta_{ec}{}^2]\right] dt \text{ and} \quad (19)$$

$$J_2 = \lim_{T\to\infty} \frac{1}{T} \mathcal{E} \int_0^T \left[[10^6\ \eta_{zcg}{}^2 + 10^6\ \eta_{zp}{}^2] + 10^{-6}\ [4\delta_{sa_c}{}^2 + 6\delta_{zc}{}^2 + 25\delta_{ec}{}^2]\right] dt \quad (20)$$

Weights were selected so as to result in significant reduction (from open loop) in vertical acceleration and wing root bending moment, without having the extrapolated control values for large gusts exceed the following limits:

$$|\delta_{sa}| < 30^{\circ}$$

$$|\delta_{z}| < 30^{\circ}$$

$$|\delta_{e}| < 10^{\circ}$$

Results were evaluated on the basis of a five second simulation run and upon the computed steady state covariances of the states and the penalized outputs.

4.3.2.2 Simulation results

Evaluation of the reduced state feedback control design procedures consisted of a series of simulation studies which considered:

- Initialization procedures
- Comparison of various feedback configurations including full state feedback controllers computed using linear optimal quadratic regulator theory.
- Effects of sensor noise
- Treatment of the C^{*} response
- Sensitivity to flight condition changes

Initialization Procedures

Because several local minimum values for the gain matrix can result when using reduced state feedback gain optimization procedures, it was of interest to examine various initialization procedures. To this effect the index J_2(penalizing vertical accelerations) was minimized for the feedback configuration $y^T = (u, \theta, n_{zcg}, q_{cg}, \alpha_v, \omega_g)$. Initial gains were selected as:

- All zero (i.e., open loop)
- Those which resulted from optimization of the configurations $y^T = (n_{zcg}, q_{cg}, \alpha_v, \omega_g)$ starting from open loop.
- Those which resulted from optimization of the configurations $y^T = (n_{zcg}, q_{cg}, \alpha_v, \omega_g, u, n_{zwt})$ with the gains on u and n_{zwt} set to zero.

Results shown in Table 3 indicate that it might be desirable to optimize from more than one initial guess and to select the most satisfactory set of gains. Consequently it was concluded that a reasonable procedure for optimizing the gains for some specified configuration might be the following:

1) First optimize the configuration consisting of only one output.
2) Use the resulting gain from step one to initialize the optimization of a configuration consisting of two outputs.
3) Continue adding one more output until the desired configuration has been optimized.
4) If desired, re-optimize this configuration with all gains initialized at zero.

Effects of Feedback Configuration

In order to examine the influence of a changing feedback structure, the two indices J_1 and J_2 defined by eqs. 19, 20 were used for designing controller based upon feedback of selected combinations of n_{zcg}, n_{zwt}, q_{cg}, α_v, ω_g, u, θ, and BMR.*

Controllers were designed based upon both an open loop initialization and upon the previously recommended procedure which involves the optimization of a sequence of an increasing number of outputs. In terms of convergence, performance index, and gust alleviation properties, the latter approach was superior. Corresponding results presented in Table 4 for designs based on minimization of J_2, which in essence penalizes vertical acceleration, indicate:

- Gust feedback is desirable for reducing the acceleration.
- Feedback of n_{zwt} in addition to n_{zcg}, ω_g, q_{cg}, and α_v significantly reduces both the vertical accelerations and the wing root bending moment.
- Little is gained and at the expense of increased controls by including the feedback of either θ, u, or BMR to the configuration defined by n_{zcg}, n_{zwt}, ω_g, q_{cg}, α_v.
- Configurations including n_{zwt} feedback show, compared with open loop, an increased damping of the lowest bending mode and an increase in the short period natural frequency.

Results shown in Table 5 for configurations designed to minimize wing root bending moment indicate:

---

* Note that in Table 2 and 3, n_{zwt} has been scaled by 10^{-2}, and BMR has been scaled by 10^{-5}. This was necessary since the initial gain perturbations performed by the program were too large in magnitude in both the positive and negative directions to indicate a meani gful search direction.

- Feedback of n_{zcg}, ω_g, q_{cg}, and α_v is almost equally effective as feedback of these quantities plus either u or θ or BMR.
- Relative small decreases in BMR sometimes are accompanied by extremely high increases in n_{zcg} and n_{zp}.
- The lowest order bending mode becomes more damped with feedback of n_{zcg}, ω_g, q_{cg}, and α_v; however a distinct short period set of poles is not distinguishable.

Simultaneous comparison of Tables 4 and 5 show that:

- Gains developed by penalizing BMR only (J_1) can result in a BMR covariance of about an order of magnitude less than that which results from penalizing the vertical accelerations (J_2). However the corresponding shear and vertical acceleration covariances (from penalizing J_1) are much larger than those corresponding to open loop operation.
- Penalizing n_{zp} and n_{zcg} (J_2) results in vertical accelerations which are an order of magnitude or more smaller than those corresponding to open loop operation, accompanied by some reduction in both BMR and S_R.
- Controls designed for penalizing BMR are larger than controls designed for penalizing the vertical acceleration.

With regard to timing information, the number of iterations required by the Zangwill-Powell procedure are shown in Table 4a. For 7 feedback outputs (i.e., 21 gains) the computational time per iteration on the CDC-7600 was about 2 minutes, for 3 feedbacks the time was about 1 min per iteration and for one feedback the time was about 1/2 minute per iteration. Thus it was not unusual to use over an hour of computing time to design a 7 output controller.

Full State Feedback Controller Comparison

As a further evaluation of the effectiveness of reduced state feedback, a comparison was made (using the 17th order model) with controllers designed using full state feedback, i.e., $u = KX$.

Note that included in this control would be direct feedback from the bending modes and actuater.

Initially the same Q and R which define J_2 were used, but this resulted in extremely large controls; consequently R was increased until the control covariances were the same order of magnitude as those in Table 2. Note for example that the first entry of Table 6 for an R of DIAG $(4,6,25) \times 10^{-2}$ results in extremely low vertical accelerations but at the expense of unacceptable control magnitude.

For the case R = DIAG (4,6,25), it should be noted that the corresponding vertical acceleration, bending moment and control covariance are within an order of magnitude of those found in Table 4 for the feedback of n_{zcg}, ω_g, q_{cg}, α_v, n_{zwt}. Thus a reduced state feedback controller if properly designed can be almost as effective as a more complex and often unrealizable full state feedback configuration.

With regard to control signal magnitudes, the gains developed from minimizing J_2 for the configuration $y^T = (n_{zcg}, \omega_g, q_{cg}, \alpha_v, n_{zwt}, u)$ gave the following maximum values over a 5 second simulation for a gust of variance $0.3183\ m^2/s^2$.

	max value in deg
δ_{sa}	1.38
δ_z	8.12
δ_e	.511

These values except for perhaps δ_z should be acceptable even for a thunderstorm situation (multiples ≈ 16). Further reduction in δ_z would be achievable by additional weighting in the performance index.

Effects of sensor noise

Tables 7a and 7b depict the effects of modelling angle of attack and pitch rate sensor noise in the design phase for the output vector $y = (n_{zcg}, q_{cg}, \alpha_v, \omega_g)^T$. In each case, the optimization procedure was initialized at open loop.

To be noted are the following results:

- Modelling of the sensor noise results in a considerable reduction in the gains which multiply q_{cg} and α_v.
- As with the sixth order model, inclusion of sensor noise results in both a distinct short period and a distinct phugoid mode in the closed loop system.
- Gains designed with sensor noise modelled, when compared with gains not incorporating sensor noise, gave a 3-4 order of magnitude reduction in the steady state covariances of n_{zcg}, n_{zp}, BMR, S_R. These covariances were compared assuming a vertical wind gust input and the feedback of the α_v and q_{cg} sensor noise components.

Design for C^* response

For additional evaluation of flight control design using the reduced state feedback computation procedures, consideration was given to the C^* response characteristic.(16) Typically a step C^* would be commanded and both feedforward and feedback gains would be designed so as to yield acceptable transient and steady state specifications. However since the gain computation programs require asymptotic stability of the augmented process (including any reference system), step inputs could not be directly accomodated in the design. Instead, gains were determined so as to transfer the C^* response from some non-zero initial value to a near zero final value.

The resulting response transient characteristics are then identical to those that would result if the objective were to regulate the difference between the actual C^* and some attainable steady state value.

Using the relationship

$$C^* = 400q_{cg} + n_{zp} \tag{4}$$

where q_{cg} is in rad/sec and n_{zp} is in g's, gains multiplying n_{zcg}, q_{cg}, and α_v were determined so as to minimize

$$J_3 = \left|\int_0^\infty [Q(C^*)^2 + 10^{-6} [4\delta_{sac}{}^2 + 6\delta_{zc}{}^2 + 25\delta_{ec}{}^2]] \, dt\right.$$

for which $Q = 10^6$ and $Q = 10^8$. Evaluation of the gains was based primarily upon a 5 second response to a unit C^* initial condition in the absence of measurement noise and disturbances.

Results shown in Table 8 and in fig. 1 indicate:

- The C^* response resulting from the reduced state feedback design settles out in about half the time and with about half the overshoot which result under open loop control.
- Both weights ($Q = 10^6$ and 10^8) give comparable results.

Additional studies showed that the C^* response corresponding to the gain matrix for the feedback configuration

$$y^T = (n_{zcg}, q_{cg}, \alpha_v, \omega_g)$$

was even worse than open loop, having an overshoot of 1.36.

Sensitivity Evaluation

Because the parameter defining the aircraft equation of motion will be changing as a function of mach number and altitude, it is important to determine the regions over the flight envelope for which a set of gains will give acceptable performance. Also of interest is any modification that can be made to the design procedure to desensitize the performance to flight condition changes. To illustrate these principles, the feedback configuration $y^T = (n_{zcg}, q_{cg}, \alpha_v, \omega_g)$ was used for designing (from open loop) gain matrices for various process representations. These gains were then evaluated using data for the cruise, climb, and landing flight conditions.

For comparison purposes, Table 9 shows the effects of gains designed for the cruise conditions evaluated at all three flight conditions. In order to attempt improvement of the behavior especially at the landing condition, various combinations of process noise (ω_p in eq. 1) and measurement noise (γ in eq. 4) were introduced into the system model. The corresponding covariances were selected to be proportional to the estimated uncertainty in each equation as follows:

$$(\dot{\omega}_{pi}^{\,2}) \approx \Sigma_j \Delta A_p^{\,2}(i,j)\ \mathcal{E}(x_p^{\,2}(j))$$

where ΔA_p was computed as the average absolute deviation in A_p over the flight conditions and $\mathcal{E}(x_p^{\,2}(j))$ was obtained as the corresponding steady state covariance in $x_p(j)$ from a typical run. Note however from Table 10b that addition of the process noise computed according to this method resulted in a degradation. In fact when combined with measurement noise, the resulting gains when applied to the landing configuration resulted in a pair of unstable eigenvalues ($+.00201 \pm j\ .144$). Reductions in the modelled noise variances (by 10^{-1} to 10^{-5}) and retention of only the measurement noise made improvements to Table 9 but nothing significantly better than the results in Table 9. Similarly various designs using the average process matrices for the 3 flight conditions did not appear to be any better than the design based upon the cruise condition without any noise.

Thus as with full state feedback design, the development of a controller which takes into account flight condition changes is somewhat of an ad hoc process, and further procedures such as those in ref. 17 should probably be considered.

5. Conclusions and Recommendations

Based upon the results presented in section 4, it can be concluded that acceptable flight controllers can indeed be designed using the developed reduced state feedback design program. Typical objectives might include the reduction of vertical accelerations and structural loads due to a gust input and the response to a C^* command.

For a given feedback configuration it is recommended that the design process consist of the optimization of a series of feedback configurations starting with only one feedback output and progressing one output at a time until the desired structure is achieved. Measurement noise should be modelled since if present but unaccounted for, severe accelerations can result.

Performance with reduced state feedback controllers can be comparable to that achievable by full state feedback systems which in reality cannot be designed because of limitations in feeding back the bending effects and in designing feedback around the actuators.

Shortcomings include the excessive computer time requirements (≈ 1 hour for 21 gains and a 17th order system), the existence of multiple minimum points, and the sensitivity to flight condition changes.

Recommendations for future consideration include:

- Develop parallel (eg. see ref. 18) rather than serial type computational algorithms, and implement the design on say the STAR computer system.
- Incorporate sensitivity penalty terms of the form $(\partial x_{pi}/\partial a_{ij})^2$ in the performance index, and consider other desensitization procedures as per ref. 17.
- Compare results with those corresponding to a full state feedback design implemented with either a linear observer or a Kalman filter.

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Appendix A Program Description

A.1 Program Name: SIRSFB

A.2 Problem solved:

Process: $\dot{x}_p = A_p x_p + B_p u_p + G_p x_n + v_p$ (A.1)

Reference system: $\dot{x}_r = A_r x_r + B_r v_r$ (A.2)

Disturbance system: $\dot{x}_n = A_n x_n + B_n v_n$ (A.3)

where x_p = (NXP x 1) plant state

x_r = (NXR x 1) reference state

x_n = (NXN x 1) external disturbance state

u_p = (NUP x 1) control vector

w_p = (NXP x 1) zero mean white plant disturbance with covariance W_p

w_r = (NWR x 1) zero mean white reference excitation noise with covariance W_r

w_n = (NXN x 1) zero mean white disturbance excitation noise with covariance W_n

Outputs:

$Z_p = D_{pp} x_p + D_{pn} x_n$ (A.4)

$Z_r = D_{rr} x_r$ (A.5)

where Z_p = (NZP x 1) plant output (A.6)

Z_r=(NZR x 1) reference output (NZR = NZP)

Control:

$u_p = K_{yp} y_p + K_{yr} y_r + K_{yn} y_n$ (A.7)

where

$y_p = C_{pp} x_p + C_{pn} x_n + \gamma$ (A.8)

$y_r = C_{rr} x_r$ (A.9)

$y_n = C_{nn} x_n$ (A.10)

y_p = (NYP x 1) plant feedback vector

y_r = (NYR x 1) reference feedforward vector

y_n = (NYN x 1) disturbance feedforward vector

γ = (NYP x 1) zero mean white sensor noise

$$\text{Index: } J = \lim_{t_f \to \infty} \mathcal{E} \int_0^{t_f} [(z_p - z_r)^T Q (z_p - z_r) + u_p^T R u_p] \, dt \qquad (A.11)$$

A.3 Program Limits

Variable	Maximum Dimensions
x_p	18
x_r	12
x_n	6
u_p	6
v_r	12
v_n	6
y_p	12
y_r	12
y_n	6

A.4 Theory

Optimal gains are determined by using the IMSL subroutine ZXPOWL which incorporates the Zangwill-Powell Search procedure.[3,4] This algorithm which does not require gradient computation is such that if the performance index were quadratic in the gains, then the search would proceed along a set of conjugate directions.

Following the reading in of the problem description the following augmented system is formed:

$$\dot{x} = A\,x + B\,u_p + v \tag{A.12}$$

$$y = C\,x + n \tag{A.13}$$

$$z = D\,x \tag{A.14}$$

$$u = K\,y \tag{A.15}$$

where

$$A = \begin{pmatrix} A_p & 0 & G_p \\ 0 & A_r & 0 \\ 0 & 0 & A_n \end{pmatrix} \tag{A.16}$$

$$B = \begin{pmatrix} B_p \\ 0 \\ 0 \end{pmatrix} \tag{A.17}$$

$$v = \begin{pmatrix} v_p \\ B_r\,v_r \\ B_n\,v_n \end{pmatrix} \tag{A.18}$$

$$C = \begin{pmatrix} C_{pp} & 0 & C_{pn} \\ 0 & C_{rr} & 0 \\ 0 & 0 & C_{nn} \end{pmatrix} \tag{A.19}$$

$$D = \begin{pmatrix} D_{pp} & 0 & D_{pn} \\ 0 & D_{rr} & 0 \end{pmatrix} \tag{A.20}$$

$$n = \begin{pmatrix} \gamma \\ 0 \\ 0 \end{pmatrix} \tag{A.21}$$

As shown in Sec. 2, the corresponding objective can be written as:

$$J = \mathrm{Trace}\,[(D^T\,Q_A\,D + C^T\,K^T\,R\,K\,C)\,P] \tag{A.22}$$

where $$Q_A = \begin{pmatrix} Q & -Q \\ -Q & Q \end{pmatrix} \tag{A.13}$$

$$K = (K_{yp}\ K_{yr}\ K_{yn}) \tag{A.24}$$

and P satisfies the steady state covariance equation

$$0 = (A - B\,K\,C)\,P + P(A - B\,K\,C)^T + B\,K\,N\,K^T\,B^T + V \tag{A.25}$$

where $$N = \mathcal{E}(n\ n^T) \tag{A.26}$$
$$V = \mathcal{E}(v\ v^T)$$

Thus given initial or perturbed values for K, the objective J can be evaluated by first solving eq. A. 25 for P and then using this value in eq. A.22. Solution of eq. A. 25 is accomplished using the method of Bartels and Stewart.(19)

For initialization it is necessary to use a gain matrix K such that all eigenvalues of $(A - B\,K\,C)$ are less than zero. If in a subsequent iteration, the gain is perturbed such that one of the eigenvalues is not negative, then the objective is arbitrarily set equal to 10^{50}, thus forcing the optimization procedure to backtrack.

A.5 Input Format

Data input consists of system dimensions, defining matrices, and various control parameters. If there is no reference system then NXR should be read as "zero" and the remaining associated data (AR, BR, CRR, DRR, VR) eliminated. Similarly if NXN = 0, then no data cards should be included for AN, BN, CPN, DPN, CNN, VN.

All matrices are stored in vector format.

Note that the initial gain matrix K must be such that $(A - \bar{B}K)$ is stable. Card format and content are shown in Table A.1.

A.6 Output

The following data is printed out:

- All input data
- Initial gains and the corresponding eigenvalues.
- Intermediate values for the objective function.
- Optimal results (preceded by "OPTIMAL INFINITE TIME SOLUTION") consisting of final gains, corresponding eigenvalues, steady-state covariance of the state x, and steady state covariance of the output z.

If the initial set of gains is such that the closed loop system is unstable, the program will attempt to find a new set of gains as discussed in Section 3.

A.7 Major Subroutines Used

READIN: Reads in problem data, prints out problem data

SETUP: Places the problem into the format described by eqs. A.12 - A.15

FUNCTION FZX(XKO): Computes the objective (FZX) given the gain matrix (XKO)

ATXPXA
SYMSLV
HSHLDR
BCKMLT
SCHUR
SYSSLV

Used in the Stewart-Bartels[19] solution of eq. A.25

EIGRF: IMSL routine used to compute eigenvalues

ZXPOWL: IMSL routine used to perform the optimization (described on next page)

Usage restricted as per letter following ZXPOWL description

```
C      SUBROUTINE ZXPOWL (F,EPS,N,X,FMIN,ITMAX,WA,IER)
C
C-ZXPOWL---------S/D-----LIBRARY 1---------------------------------------
C
C   FUNCTION            - POWELL'S ALGORITHM TO FIND A (LOCAL) MINIMUM
C                           OF A REAL FUNCTION OF N REAL VARIABLES
C   USAGE               - CALL ZXPOWL (F,EPS,N,X,FMIN,ITMAX,WA,IER)
C   PARAMETERS   F      - A FUNCTION SUBPROGRAM WRITTEN BY THE USER
C                EPS    - CONVERGENCE CRITERION - SEE ELEMENT
C                           DOCUMENTATION
C                N      - LENGTH OF THE VECTOR ARRAY X (INPUT)
C                X      - A VECTOR ARRAY OF LENGTH N. ON INPUT, X IS AN
C                           INITIAL GUESS FOR THE MINIMUM. ON OUTPUT
C                           X IS THE COMPUTED MINIMUM POINT
C                FMIN   - F(X) - FUNCTION F EVALUATED AT X (OUTPUT)
C                ITMAX  - ON INPUT = THE MAXIMUM ALLOWABLE NUMBER OF
C                           ITERATIONS PER ROOT AND ON OUTPUT = THE
C                           NUMBER OF ITERATIONS USED
C                WA     - A VECTOR WORK AREA OF LENGTH N*(N+4)
C                IER    - ERROR PARAMETER (OUTPUT)
C                         TERMINAL ERROR = 128+N
C                           N = 1 NO FINITE MINIMUM OBTAINED
C                           N = 2 F IS LEVEL ALONG A LINE THROUGH X
C                           N = 4 FAILURE TO CONVERGE IN ITMAX
C                                  ITERATIONS
C                           N = 8 GRADIENT 'LARGE' AT CALCULATED MINIMUM
C   PRECISION           - SINGLE/DOUBLE
C   REQ'D IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
```

CALL ZXPOWL(F,EPS,N,X,FMIN,ITMAX,WA,IER)

Purpose

This routine uses Powell's algorithm to find a local minimum of a real function of N real variables.

Algorithm

Let $F(X_1,X_2,\ldots,X_N)$ be a function of N real variables $X_1,X_2,\ldots,X_N$. ZXPOWL seeks a point $X^*=(X_1^*,X_2^*,\ldots,X_N^*)^T$ which furnishes a local minimum to the function F at X*, i.e.,

$F(X^*)=\min(F(X))$. X in S where S is an open set in E^N.

Note that there are no side constraints so that the problem is simply an unconstrained minimization.

ZXPOWL uses Zangwill's modification of Powell's conjugate direction algorithm to perform the minimization. The algorithm has the notable feature that it will minimize a quadratic form in a finite number of steps.

See references: Zangwill, W., (1967) "Minimizing a function without calculating derivatives", Computer Journal, Vol. 10, pp. 293-296.

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Programming Notes

The user must furnish the function F as an EXTERNAL FUNCTION subprogram F(X) where X is an N-vector of coordinate abscissa. The user must not alter the values in X.

Convergence of the algorithm is defined as

$$(1) \quad \left| \frac{F(X^m)-(F(X^{m-1})}{MAX[1.,|F(X^{m-1})|]} \right| \text{ less than EPS}$$

If an X^m is found within ITMAX iterations satisfying (1) X^m is accepted as the problem solution, and returned to the user. Also the approximate minimum of F, i.e., $F(X^m)$, is returned in the output parameter FMIN. Here X^m denotes the value of X at the m-th iteration.

Accuracy

Let F(X)=SIN(X)+COS(X) and $G(Y)=3Y^3+5Y^2+Y+4$. Setting $X_0=3$, $Y_0=1$, and $EPS=10^{-6}$, ZXPOWL obtains values given in the following table:

Function	Exact Solution	ZXPOWL	Number of Iterations
F	3.926991	3.927358	2
G	-0.1111111	-0.1111725	4

Note that (1) does not guarantee X^m to be close to X*. It does imply for "well-behaved" functions that $F(X^m)$ is close to F(X*).

March 16, 1976

Professor Howard Kaufman
Associate Professor
Electrical and Systems
Engineering Department
Rensselaer Polytechnic Institute
Troy, NY 12181

Dear Professor Kaufman

Thank you for your letter of March 10, 1976. IMSL grants permission for Rensselaer Polytechnic to use IMSL subroutines EIGRF, ZXPOWL, and ZXMIN as part of an application package being developed for NASA-Langley. IMSL requests, if possible, that Rensselaer Polytechnic only make the object code available to NASA-Langley. We also ask that NASA-Langley be informed that these routines are proprietary and may only be used as part of the application package for which they were developed.

Please let us know if we may be of any further service.

Best regards

L. L. Williams

L. L. Williams
Director, Operations

maa

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Table A.1

Note: Cards for AR, BR, CRR, DRR, VR can be eliminated if NXR=0.

Cards for AN, BN, CPN, DPN, CNN, VN, GP can be eliminated if NXN = 0.

Cards for VP should be eliminated if NVP = 0.

Input Format for Program SIRSFB

Card Number	Column Number	Description	Format
1	10	NXP = Dimension of x_p	8I10
	20	NXR = Dimension of x_r	
	30	NXN = Dimension of x_n	
	40	NVP = Dimension of v_p	
	50	NVR = Dimension of v_r	
	60	NVN = Dimension of v_n	
	70	NUP = Dimension of u_p	
2	10	NYP = Dimension of y_p	8I10
	20	NYR = Dimension of y_r	
	30	NYN = Dimension of y_n	
	40	NZP = Dimension of z_p	
	50	NZR = Dimension of z_r	
	60	ITMAX = Maximum No. of iterations	
3 + i	1	AP (i, 1) eq. A.1	8E10.4
	11	AP (i, 2)	
	etc.	...	
3 + UXP	1	AP (NXP, 1)	8E10.4
	11	AP (NXP, 2)	
	etc.	...	
2 + NXP + i	1	BP (i, 1) eq. A.1	8E10.4
	11	BP (i, 2)	
	etc.	...	
3 + 2 NXP	1	BP (NXP, 1)	8E10.4
	11	BP (NXP, 2)	
	etc.	...	

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	Card Number	Column Number	Description		Format
	3 + 2 NXP + i	1 11 etc.	CPP (i, 1) CPP (i, 2) ...	eq. A.8	8E10.4
	3 + 2 NXP + NYP	1 11 etc.	CPP (NYP, 1) CPP (NYP, 2) ...		8E.10.4
	3 + 2 NXP + NYP + i	1 11 etc.	DPP (i, 1) DPP (i, 2) ...	eq. A.4	8E10.4
	3 + 2 NXP + NYP + NZP	1 11 etc.	DPP (NZP, 1) DPP (NZP, 2) ...		8E10.4
*	4 + 2 NXP + NYP + NZP = NP	1 11	$\mathcal{E}(V_p^2(1)) = V_p(1)$ $\mathcal{E}(V_p^2(2)) = V_p(2)$		8E10.4
†	NP + i	1 11 etc.	AR (i, 1) AR (i, 2) ...	eq. A.2	8E10.4
†	NP + NXR	1 11 etc.	AR (NXR, 1) AR (NXR, 2)		8E10.4
†	NP + NXR + i	1 11 etc.	BR (i, 1) BR (i, 2) ...	eq. A.2	8E10.4
†	NP + 2 NXR	1 11 etc.	BR (NXR, 1) BR (NXR, 2) ...		8E10.4

* Only if NVP ≠ 0

† Only if NXR ≠ 0

Card Number	Column Number	Description	Format
NP + 2 NXR + i †	1 11 etc.	CRR (i, 1) eq. A.9 CRR (i, 2) ...	8E10.4
NP + 2 NXR + NYR †	1 11 etc.	CRR (NYR, 1) CRR (NYR, 2) ...	8E10.4
NP + 2 NYR + NYR + i †	1 11 etc.	DRR (i, 1) eq. A.5 DRR (i, 2) ...	8E10.4
NP + 2 NXR + NYR + NZR †	1 11 etc.	DRR (NZR, 1) DRR (NZR, 2) ...	8E10.4
NP + 2NXR + NYR + NZR + 1 = NPR †	1 11 etc.	VR (1) = $\mathcal{E}(v_r^2(1))$ VR (2) = $\mathcal{E}(v_r^2(2))$	8E10.4
NPR + i **	1 11 etc.	AN (i, 1) eq. A.3 AN (i, 2) ...	8E10.4
NPR + NXN **	1 11 etc.	AN (NXN, 1) AN (NXN, 2) ...	8E10.4
NPR + NXN + i **	1 11 etc.	BN (i, 1) eq. A.3 BN (i, 2) ...	8E10.4
NPR + 2 NXN **	1 11 etc.	BN (NXN, 1) BN (NXN, 2) ...	8E10.4

† Only if NXR $\neq$ 0

** Only if NXN $\neq$ 0

Page 4 of Table A.1

Card Number	Column Number	Description	Format
NPR + 2 NXN + i **	1 11 etc.	CPN (i, 1) eq. A.8 CPN (i, 2) ...	8E10.4
NPR + 2 NXN + NYP **	1 11 etc.	CPN (NYP, 1) CPN (NYP, 2) ...	8E10.4
NPR + 2 NXN + NYP + i **	1 11 etc.	DPN (i, 1) eq. A.4 DPN (i, 2) ...	8E10.4
NPR + 2 NXN + 2 NYP + NZP **	1 11 etc.	DPN (NZP, 1) DPN (NZP, 2) ...	8E10.4
NPR + 2 NXN + 2 NYP + NZP + i ††	1 11 etc.	CNN (i, 1) eq. A.10 CNN (i, 2) ...	8E10.4
NPR + 2 NXN + 2 NYP + NZP + NYN ††	1 11 etc.	CNN (NYN, 1) CNN (NYN, 2) ...	8E10.4
NPR + 2 NXN + 2 NYP + NZP + NYN + 1 **	1 11 etc.	VN (1) = $\mathcal{E}(v_n^{\,2}(1))$ VN (2) = $\mathcal{E}(v_n^{\,2}(2))$ ...	8E10.4
NPR + 2NXN + 2NYP + NZP + NYN + 1+i **	1 11 etc.	GP (i, 1) eq. A.1 GP (i, 2) ...	8E10.4
NPR + 2NXN + 2NYP + NZP + NYN+1+NXP = NPRN **	1 11 etc.	GP (NXP, 1) GP (NXP, 2) ...	8E10.4

** Only if NXN $\neq$ 0

Only if NYN $\neq$ 0

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Card Number	Column Number	Description		Format
NPRN + i	1 11 etc.	R (i, 1) R (i, 2) ...	eq. A.11	8E10.4
NPRN + NUP	1 11 etc.	R (NUP, 1) R (NUP, 2) ...		8E10.4
NPRN + NUP + i	1 11 etc.	Q (i, 1) Q (i, 2) ...	eq. A.11	8E10.4
NPRN + NUP + NZP	1 11 etc.	Q (NZP, 1) Q (NZP, 2) ...		8E10.4
NPRN + NUP + NZP + i	1 11 etc.	WW (i, 1) WW (i, 2) ...	CoV of γ in eq. A.8	8E10.4
NPRN + NUP + NZP + NYP	1 11 etc.	WW (NYP, 1) WW (NYP, 2) ...		8E10.4
NPRN + NUP + NZP + NYP + i	1 11 etc.	XKO (i, 1) XKO (i, 2) ...	Initial value of K in eq. A.24	8E10.4
NPRN + 2 NUP + NZP + NYP	1 11 etc.	XKO (NUP, 1) XKO (NUP, 2) ...		8E10.4

Appendix B

TIFS DATA (from ref. 14)

$$x^T = (u,\ \omega,\ q,\ \theta,\ n_1,\ \dot{n}_1,\ n_2,\ \dot{n}_2,\ n_3,\ \dot{n}_3,\ n_4,\ \dot{n}_4,\ n_5,\ \dot{n}_5)$$

$$u^T = (\delta_{sa},\ \delta_z,\ \delta_e,\ \omega_g)$$

$$y^T = (n_{zp},\ n_{zcg},\ n_{zt},\ n_{z\omega t},\ n_{zFSF},\ n_{zast},\ n_{zRHT},\ q_{cg},\ \alpha_v,\ S_R,\ B_{MR},\ T_R)$$

See pages 21, 22 for definitions

LANDING

$$\dot{x} = F'x + G'u$$
$$y = Ax + Bu$$

$F' =$

COLUMNS 1 THRU 10

9.93275D-03	1.57518D-01	-8.33021D+00	-9.73273D+00	-1.66949D-03	-1.91982D-03	1.57614D-02	9.20956D-04	3.30895D-02	-1.49457D-04
-3.33360D-01	-9.48826D-01	6.77015D+01	-1.20088D+00	3.09811D-01	1.64165D-02	-6.59667D-01	-7.53406D-03	1.67889D-01	1.23892D-02
-4.65027D-03	-2.76522D-02	-1.30033D+00	5.60551D-03	1.48637D-02	1.39117D-04	-6.91304D-02	-2.10375D-03	1.83709D-01	2.22356D-03
0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0
6.58945D+01	1.65250D+02	-4.46970D+02	8.79598D+00	-4.70632D+02	-6.84075D+00	9.17549D+01	1.52595D+00	1.37311D+02	-1.71880D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0
-1.92843D+01	-6.72146D+01	-5.12508D+02	1.08663D+00	1.77444D+01	2.08577D+00	-1.03898D+03	-3.58897D+00	6.66971D+01	2.02753D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00
1.05126D+01	6.62295D+01	8.35662D+02	-2.01600D+00	-1.40282D+01	-1.13687D+00	6.21987D+01	2.49482D+00	-1.81025D+03	-5.42779D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.07658D+00	4.41861D+01	1.19348D+03	-4.43325D+00	4.04251D+00	1.36891D+00	4.66683D+01	2.14664D+00	-2.11125D+02	-2.88589D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.31564D+01	-3.62138D+01	-6.41572D+01	-8.17948D-01	8.26877D+00	2.39580D+00	-1.97382D+01	-1.37670D+00	-6.26609D+00	1.45467D+00

COLUMNS 11 THRU 14

-1.40585D-02	-1.93984D-05	1.40702D-02	3.04849D-04
6.22008D-01	2.43241D-03	-2.71257D-01	-9.47379D-04
3.80552D-02	2.43507D-03	2.41196D-02	2.98622D-04
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-9.37400D+01	2.41773D+00	1.25416D+02	2.46718D+00
0.0	0.0	0.0	0.0
3.77405D+01	1.14424D+00	-5.00320D+01	-1.53937D+00
0.0	0.0	0.0	0.0
-2.36123D+01	-3.95329D+00	1.04640D+02	1.44632D+00
0.0	1.00000D+00	0.0	0.0
-2.17379D+03	-1.01002D+01	1.64210D+02	1.59620D-01
0.0	0.0	0.0	1.00000D+00
1.28705D+01	-4.67407D-01	-3.76199D+03	-6.52659D+00

$G' =$

7.20180D-03	-5.49812D-03	2.53231D-02	8.19753D-02
-1.74570D-01	-1.14129D-01	-1.08501D-01	-1.00667D+00
-9.25392D-03	-3.44984D-03	-4.64568D-02	-3.67438D-02
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
6.19076D+01	4.48580D+01	-1.25765D+01	1.65808D+02
0.0	0.0	0.0	0.0
-3.06521D+01	-5.77638D+00	-3.05168D+01	-6.72054D+01
0.0	0.0	0.0	0.0
1.80412D+01	-2.06483D+01	5.18845D+01	6.61807D+01
0.0	0.0	0.0	0.0
-5.17137D+00	-9.96016D+00	5.81485D+01	4.40231D+01
0.0	0.0	0.0	0.0
-3.58557D+01	1.92757D+01	1.02734D+00	-3.62789D+01

A =

COLUMNS 1 THRU 10

-3.36266D-03	-3.24614D-02	3.87664D-01	1.70890D-03	-3.37891D-01	-7.27082D-04	-1.72578D+00	-2.68590D-03	5.72064D-01	7.87839D-04
-1.29774D-02	-2.56780D-02	1.25603D-01	1.20425D-03	-1.20320D-01	-3.91930D-04	2.14306D-01	6.04439D-04	-1.10665D+00	-1.47710D-03
8.68896D-05	-3.89639D-02	-1.72598D+00	1.18391D-02	-3.23121D-01	-7.23709D-04	-1.36242D+00	-4.64966D-03	1.49378D+00	4.28875D-03
-6.77892D-01	-1.82415D+00	1.29545D+00	-6.78253D-02	3.31985D+00	8.30288D-02	-4.88355D+00	-3.79243D-02	3.22142D+00	3.96603D-02
-1.53794D-01	-4.08550D-01	5.92532D-01	-1.60449D-02	9.85487D-01	1.22641D-02	-5.71940D-01	-3.25402D-03	2.42241D-01	4.27450D-03
-1.65207D-01	-4.31286D-01	4.89357D-01	-1.61560D-02	1.03248D+00	1.31188D-02	-9.28125D-01	-4.24711D-03	-7.90202D-01	3.17736D-03
-2.84285D-01	-2.08413D+00	-4.17091D+01	1.45289D-01	-4.14012D-01	-9.48774D-03	-5.77676D+00	-9.82205D-02	2.75953D+01	1.45711D-01
8.88181D-04	2.30769D-03	5.72646D+01	1.24615D-03	-4.88684D-03	-2.30582D-03	5.89003D-03	-9.44544D-04	-5.82165D-03	4.30081D-02
3.48911D-04	8.43014D-01	-8.55846D+00	8.48871D-04	-1.74152D-03	6.44973D-03	2.09822D-03	1.32381D-02	-3.50590D-03	-2.34998D-03

COLUMNS 11 THRU 14

-9.43269D-01	-1.25885D-03	1.99165D+00	1.94808D-03
7.13556D-01	8.93315D-04	-2.60776D+00	-2.68523D-03
-2.59845D+00	-2.52790D-03	3.07782D+00	4.62224D-03
-4.66810D+00	-3.19814D-02	-3.53078D+01	-8.51050D-02
3.06595D-01	-2.21388D-03	6.40411D+00	5.67697D-03
9.62577D-01	-1.40296D-03	5.55305D+00	4.31219D-03
4.98933D+01	2.87391D-01	-1.66024D+01	-4.35209D-02
1.41088D-02	-1.48411D-03	2.34414D-03	1.74050D-02
-7.35924D-04	3.13101D-03	5.94949D-04	-3.49878D-03

B =

5.03505D-05	1.12642D-02	-1.69391D-02	-2.80271D-02
-3.21095D-03	6.38612D-03	1.05810D-02	-3.10913D-02
-6.47459D-03	3.09992D-03	-8.33593D-02	-5.54997D-02
-9.29984D-01	-1.28299D-01	-1.59835D-02	-1.83539D+00
-9.41762D-02	-1.33825D-01	-1.15460D-02	-4.15336D-01
-1.07240D-01	-1.46207D-01	-1.00247D-02	-4.38964D-01
-2.30070D-01	5.61468D-01	-2.15266D+00	-2.09634D+00
9.54183D-04	4.84798D-04	-5.63241D-05	2.35702D-03
3.34375D-04	1.47574D-04	1.69095D-04	8.43040D-01

CLIMB

$$\dot{x} = F'x + G'u$$

$$y = Ax + Bu$$

$F' =$

COLUMNS 1 THRU 10

4.83946D-03	1.18872D-01	-6.64386D+00	-9.77339D+00	-1.64398D-03	-1.50359D-03	1.44961D-02	6.99335D-04	3.19649D-02	-1.00546D-04
-2.96828D-01	-1.00113D+00	8.26434D+01	-7.99181D-01	2.87577D-01	1.72850D-02	-6.91405D-01	-7.81327D-03	9.64184D-02	1.35125D-02
-3.26716D-03	-2.89495D-02	-1.20441D+00	2.90302D-03	1.10203D-02	1.07093D-04	-6.96119D-02	-2.21272D-03	1.83111D-01	2.29933D-03
0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0
5.49991D+01	1.76688D+02	-5.05739D+02	5.32728D+00	-4.70876D+02	-7.23664D+00	9.88991D+01	1.56314D+00	1.58687D+02	-1.91414D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0
-1.55224D+01	-7.20721D+01	-5.50597D+02	6.32244D-01	1.63262D+01	2.21523D+00	-1.04280D+03	-3.71373D+00	6.34474D+01	2.16272D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00
6.76404D+00	7.14779D+01	9.07068D+02	-1.21122D+00	-1.14069D+01	-1.19320D+00	6.62993D+01	2.66559D+00	-1.81508D+03	-5.64612D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.35813D+00	4.73956D+01	1.30286D+03	-2.69096D+00	7.27220D+00	1.52054D+00	4.91979D+01	2.30653D+00	-2.26887D+02	-3.03986D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1.08796D+01	-3.94023D+01	-6.53759D+01	-5.17440D-01	8.92657D+00	2.58293D+00	-2.30828D+01	-1.47193D+00	-8.59925D+00	1.51194D+00

COLUMNS 11 THRU 14

-1.23991D-02	3.47460D-05	1.34128D-02	2.34088D-04
6.84508D-01	2.53862D-03	-2.90951D-01	-6.90279D-04
5.29748D-02	2.61034D-03	2.29226D-02	3.34447D-04
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-9.42441D+01	2.67508D+00	1.33875D+02	2.59760D+00
0.0	0.0	0.0	0.0
4.92448D+01	1.22756D+00	-5.27647D+01	-1.62888D+00
0.0	0.0	0.0	0.0
-4.88408D+01	-4.28042D+00	9.85194D+01	1.51804D+00
0.0	1.00000D+00	0.0	0.0
-2.21225D+03	-1.07668D+01	1.47142D+02	1.21132D-01
0.0	0.0	0.0	1.00000D+00
1.63656D+01	-5.12963D-01	-3.77494D+03	-6.77946D+00

$G' =$

5.30315D-03	1.46026D-02	-3.80145D-02	6.20502D-02
-2.21563D-01	-2.56935D-01	-1.70899D-01	-1.08144D+00
-1.13881D-02	3.41875D-03	-6.15575D-02	-3.90630D-02
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
7.92693D+01	5.81930D+01	-1.62711D+01	1.77357D+02
0.0	0.0	0.0	0.0
-3.92217D+01	-7.07899D+00	-3.92135D+01	-7.20530D+01
0.0	0.0	0.0	0.0
2.28388D+01	-2.72788D+01	6.68445D+01	7.14065D+01
0.0	0.0	0.0	0.0
-7.17672D+00	-1.35064D+01	7.49956D+01	4.71784D+01
0.0	0.0	0.0	0.0
-4.63835D+01	2.50370D+01	1.34327D+00	-3.94810D+01

CLIMB (Cont.)

A =

COLUMNS 1 THRU 10

-4.17948D-03	-3.40552D-02	2.53522D-01	9.31978D-04	-3.38479D-01	-7.15330D-04	-1.72671D+00	-2.71453D-03	5.74746D-01	8.32016D-04
-1.38118D-02	-2.61433D-02	1.83213D-01	1.52675D-04	-1.21012D-01	-4.06067D-04	2.13909D-01	6.16439D-04	-1.10601D+00	-1.46215D-03
-2.30662D-04	-3.94103D-02	-1.56326D+00	5.94610D-03	-3.30516D-01	-7.66996D-04	-1.36106D+00	-4.78106D-03	1.48642D+00	4.36216D-03
-5.62201D-01	-1.95685D+00	1.70286D+00	-4.20523D-02	3.32088D+00	8.84491D-02	-4.98134D+00	-3.95495D-02	3.00867D+00	4.28932D-02
-1.29727D-01	-4.34536D-01	7.37742D-01	-1.02764D-02	9.81370D-01	1.28799D-02	-5.86438D-01	-3.27739D-03	1.95293D-01	4.66330D-03
-1.39896D-01	-4.58601D-01	6.47981D-01	-1.03953D-02	1.02835D+00	1.37763D-02	-9.43444D-01	-4.28849D-03	-8.38839D-01	3.57760D-03
-1.87500D-01	-2.23982D+00	-4.51046D+01	8.66256D-02	-5.30558D-01	-1.16773D-02	-5.90139D+00	-1.04782D-01	2.80122D+01	1.52682D-01
7.27722D-04	2.41978D-03	5.72581D+01	9.32552D-04	-4.77759D-03	-2.30888D-03	5.86719D-03	-9.65027D-04	-6.22396D-03	4.29762D-02
2.40098D-04	6.96109D-01	-7.06927D+00	5.45871D-04	-1.41943D-03	5.32377D-03	1.76445D-03	1.09243D-02	-3.16689D-03	-1.94891D-03

COLUMNS 11 THRU 14

-9.48095D-01	-1.25709D-03	1.99527D+00	1.98355D-03
7.13850D-01	9.17354D-04	-2.61225D+00	-2.71751D-03
-2.59634D+00	-2.54429D-03	3.06795D+00	4.79159D-03
-4.61953D+00	-3.46841D-02	-3.55292D+01	-8.88497D-02
3.19579D-01	-2.51779D-03	6.40548D+00	5.83634D-03
9.76989D-01	-1.70433D-03	5.55620D+00	4.43673D-03
5.11316D+01	3.07190D-01	-1.60545D+01	-4.43608D-02
1.51001D-02	-1.47140D-03	1.64397D-03	1.73940D-02
-7.27742D-04	2.59576D-03	3.45892D-04	-2.89188D-03

B =

-6.15895D-05	-4.44303D-03	-2.28761D-02	-3.06989D-02
-4.26021D-03	-3.32091D-03	1.04795D-02	-3.37821D-02
-7.58112D-03	2.61710D-03	-1.12326D-01	-5.94040D-02
-1.19534D+00	-1.75322D-01	-2.25920D-02	-1.97138D+00
-1.19501D-01	-1.84553D-01	-1.78917D-02	-4.43791D-01
-1.36234D-01	-1.99084D-01	-1.60632D-02	-4.68852D-01
-3.34139D-01	7.47823D-01	-2.77932D+00	-2.25393D+00
1.20331D-03	6.41972D-04	-5.27562D-05	2.47926D-03
3.53390D-04	1.65637D-04	2.05457D-04	6.96135D-01

CRUISE

$$\dot{x} = F'x + G'u$$

$$y = Ax + Bu$$

$F' =$

COLUMNS 1 THRU 10

-1.64315D-02	-1.02562D-03	1.46698D+00	-9.80567D+00	-1.35713D-04	-4.07826D-04	1.61219D-03	1.24626D-04	2.03893D-02	6.91449D-05
-1.51308D-01	-1.89056D+00	1.48718D+02	9.72476D-02	3.70574D-01	3.13287D-02	-1.42953D+00	-1.15276D-02	-3.38125D-01	2.80295D-02
3.32996D-04	-5.42699D-02	-2.14111D+00	-2.87042D-04	-4.23662D-03	2.36282D-04	-1.38917D-01	-4.04582D-03	3.50836D-01	3.78983D-03
0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0	0.0	0.0
3.16982D+01	3.29181D+02	-1.11489D+03	-7.74367D-01	-5.15583D+02	-1.22902D+01	2.36147D+02	2.13940D+00	4.24950D+02	-4.84081D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00	0.0	0.0
-5.77646D+00	-1.38574D+02	-9.96042D+02	-4.55916D-02	2.17824D+01	4.03007D+00	-1.12078D+03	-5.28541D+00	9.96881D+01	4.05593D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00000D+00
-6.40497D+00	1.41659D+02	1.74399D+03	1.43656D-01	-4.83939D+00	-2.26442D+00	1.58236D+02	4.82396D+00	-1.97501D+03	-8.44703D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-4.08697D+00	8.97408D+01	2.58232D+03	3.80319D-01	3.47086D+01	2.68477D+00	1.14033D+02	4.45728D+00	-5.33345D+02	-4.91097D+00
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-4.40977D+00	-8.13931D+01	-7.35004D+01	1.02268D-01	2.17929D+01	4.68764D+00	-6.93920D+01	-2.44697D+00	-3.49967D+01	3.09367D+00

COLUMNS 11 THRU 14

4.04059D-03	1.98819D-04	5.79081D-03	2.80099D-05
1.64492D+00	2.04802D-03	-7.83171D-01	-1.46271D-03
2.13125D-01	5.12616D-03	1.86397D-02	3.14423D-04
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-1.80990D+02	5.59853D+00	3.07937D+02	4.21035D+00
0.0	0.0	0.0	0.0
1.77306D+02	2.20924D+00	-1.21130D+02	-2.81123D+00
0.0	0.0	0.0	0.0
-2.73342D+02	-7.70082D+00	1.43896D+02	2.68683D+00
0.0	1.00000D+00	0.0	0.0
-2.49575D+03	-1.76586D+01	1.08442D+02	3.70107D-01
0.0	0.0	0.0	1.00000D+00
5.90178D+01	-1.04689D+00	-3.86286D+03	-9.21408D+00

$G' =$

-1.15492D-02	1.64642D-03	-1.51101D-02	7.96913D-03
-6.63975D-01	-6.67714D-01	-7.93065D-01	-1.91403D+00
-3.60771D-02	5.19263D-03	-1.92570D-01	-7.91176D-02
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
2.35992D+02	1.72348D+02	-4.38806D+01	3.29756D+02
0.0	0.0	0.0	0.0
-1.17499D+02	-1.58064D+01	-1.15480D+02	-1.38724D+02
0.0	0.0	0.0	0.0
7.05093D+01	-9.12345D+01	1.96205D+02	1.41824D+02
0.0	0.0	0.0	0.0
-1.72323D+01	-5.23927D+01	2.18796D+02	8.98583D+01
0.0	0.0	0.0	0.0
-1.37602D+02	7.37373D+01	3.23767D+00	-8.15131D+01

A =

COLUMNS 1 THRU 10

2.98313D-03	-6.78004D-02	1.92844D-01	-2.78703D-04	-3.37656D-01	-6.67064D-04	-1.70885D+00	-2.50120D-03	5.69931D-01	8.19843D-04
-1.05415D-07	-5.04585D-02	1.48908D-01	-1.12024D-04	-1.22199D-01	-5.35827D-04	2.10833D-01	9.17269D-04	-1.10781D+00	-1.56534D-03
7.91893D-03	-7.69430D-02	-2.88543D+00	-7.22244D-04	-3.52653D-01	-6.45506D-04	-1.43641D+00	-6.89204D-03	1.67663D+00	5.96560D-03
-2.81495D-01	-3.75397D+00	4.99975D+00	6.52609D-03	3.81625D+00	1.55015D-01	-6.74407D+00	-5.84984D-02	7.15021D-01	8.68275D-02
-7.23168D-02	-8.01842D-01	1.47388D+00	1.39175D-03	1.05749D+00	2.16907D-02	-9.11803D-01	-4.20574D-03	-2.91680D-01	1.07766D-02
-8.12146D-02	-8.45101D-01	1.34575D+00	1.42746D-03	1.10835D+00	2.32089D-02	-1.28714D+00	-5.50140D-03	-1.33652D+00	9.94016D-03
1.63598D-01	-4.33669D+00	-8.79136D+01	-1.15655D-02	-1.24327D+00	-1.70521D-02	-8.87651D+00	-1.90089D-01	3.74009D+01	2.39685D-01
4.91329D-04	6.12084D-03	5.72003D+01	2.85890D-04	-7.18468D-03	-2.42975D-03	1.01051D-02	-1.12338D-03	-7.89313D-03	4.26775D-02
8.05567D-05	3.83791D-01	-3.90033D+00	9.93001D-05	-1.16868D-03	2.90808D-03	1.94911D-03	5.97230D-03	-3.31456D-03	-1.12318D-03
-6.61382D+02	3.00109D+02	1.32090D+03	3.96158D-01	1.42918D+04	4.34255D+01	2.79422D+04	4.89556D+01	1.54306D+04	3.01873D+01
-6.70137D+03	-1.26419D+03	-4.82182D+02	2.43285D+00	1.07680D+05	2.42627D+02	1.39556D+04	5.25686D+01	2.81485D+04	2.21592D+01
7.45964D+02	4.08637D+03	-1.18501D+04	-9.58347D+00	-2.73565D+03	-4.26228D+01	5.06360D+04	5.41788D+01	9.95598D+04	4.98098D+01

COLUMNS 11 THRU 14

-9.92391D-01	-1.78706D-03	2.01600D+00	2.27375D-03
7.16798D-01	9.08689D-04	-2.61060D+00	-3.03900D-03
-2.48517D+00	-1.67172D-03	3.09096D+00	5.99318D-03
-3.23890D+00	-6.42781D-02	-3.79602D+01	-1.28507D-01
6.28847D-01	-6.17203D-03	6.11288D+00	6.10717D-03
1.30411D+00	-5.35481D-03	5.23658D+00	4.23168D-03
6.08689D+01	5.13822D-01	-1.60739D+01	-7.37336D-02
2.37003D-02	-1.30179D-03	1.65499D-03	1.73437D-02
-1.48592D-03	1.48220D-03	4.17959D-04	-1.60056D-03
5.16862D+03	5.92657D+00	6.32212D+04	8.64743D+01
2.04366D+04	4.23489D+01	2.31690D+05	2.82758D+02
-5.73164D+04	-3.74742D+01	5.01225D+03	-1.99674D+01

B =

3.09082D-03	8.74805D-03	-7.73345D-02	-4.29274D-02
-1.19731D-02	-4.50435D-03	2.87755D-03	-5.17204D-02
-2.21477D-02	9.74712D-03	-3.67681D-01	-1.09014D-01
-3.55174D+00	-5.01447D-01	-1.33488D-01	-3.76416D+00
-3.58480D-01	-5.31642D-01	-8.89951D-02	-8.04988D-01
-4.08432D-01	-5.78442D-01	-8.47610D-02	-8.50620D-01
-1.11834D+00	2.63632D+00	-8.14294D+00	-4.37454D+00
4.77344D-03	2.42134D-03	1.26000D-04	6.20592D-03
8.27101D-04	3.04879D-04	6.36364D-04	3.83812D-01
4.57079D+02	1.60496D+02	2.48589D+03	-1.17895D+04
1.82384D+03	-2.47807D+03	9.47424D+03	-1.22727D+05
2.12284D+02	2.24175D+02	-4.02399D+02	1.54896D+03

Appendix C

Publications and Presentations

Presentations which incorporate a publication in a preprint volume.

"Computation of Output Feedback Gains for Linear Stochastic Systems Using the Zangwill-Powell Method", 1977 JACC, San Francisco, CA, pp. 1576-1581.

"Application of Stochastic Optimal Reduced State Feedback Gain Computation Procedures to the Design of Aircraft Gust Alleviation Controllers",IFAC VII World Congress, Helsinki, June 1978.

"Application of Stochastic Optimal Reduced State Feedback to a 17th Order Aircraft Model," "Optimization Days, 1978," Montreal, May 1978 (Abstract published).

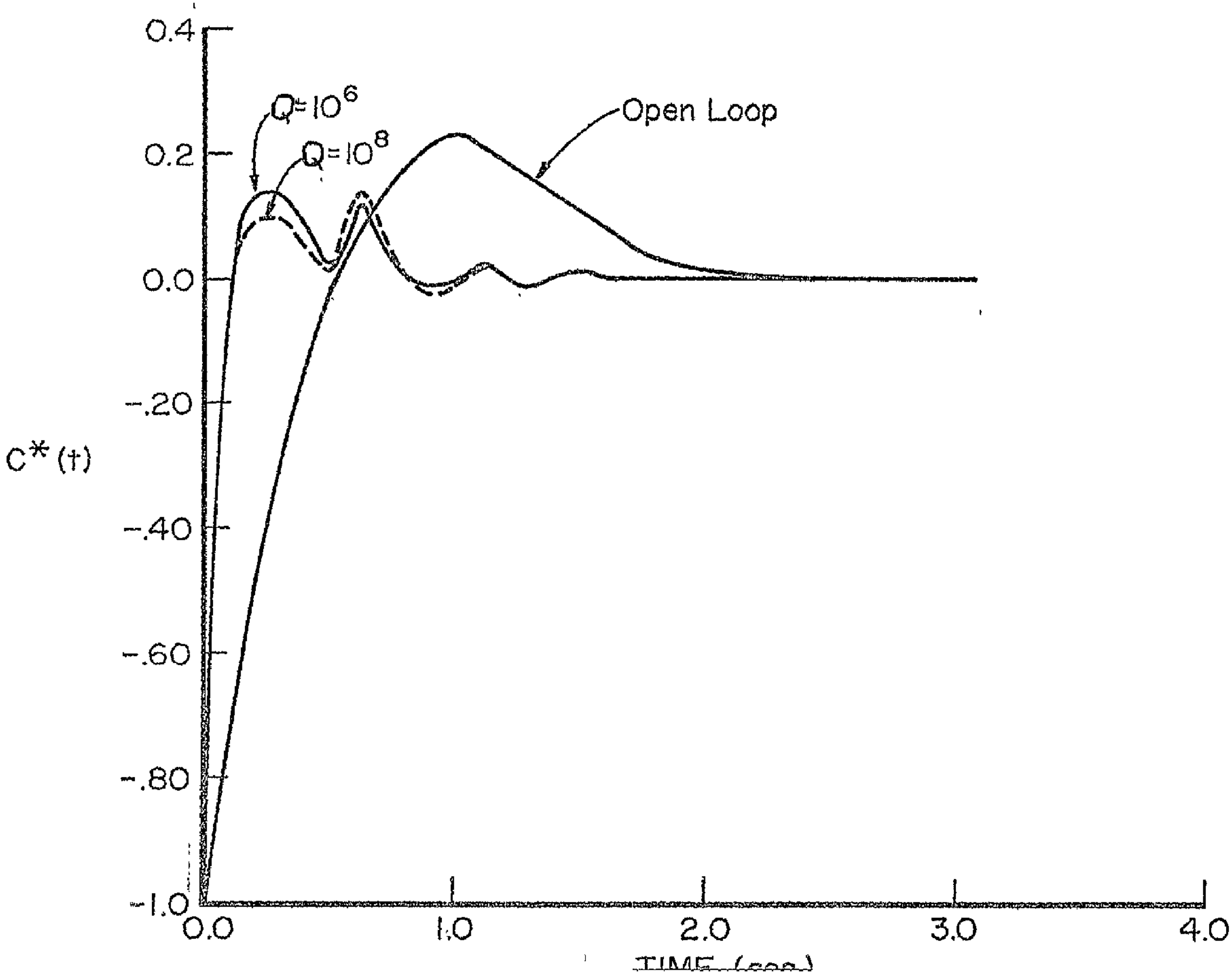
Relative Master Project Reports.

"Comparison of Stabilizing Subroutines",RPI, ESE Dept., P. Baratta, Aug. 1976.

"Reduced State Feedback Gain Computation Using the Sequential Unconstrained Minimization Technique", RPI, ESE Dept., J. Yip, June 1977.

"Investigation of the Design and Performance of Reduced State Feedback Controllers",RPI, ESE Dept., K. Sobel, May 1977.

Fig. I C^* Responses



58.

Table 1

Effect of various controller configurations

Initial gains set equal to zero, sensor noise not accounted for in design

Feedback Configuration	gain matrix	eigenvalues	J*	Vertical Acceleration rms values (ft/sec^2), No sensor noise n_{z1}	n_{z2}	w. sensor noise n_{z1}	n_{z2}
Open loop	[0]	$-1.45 \pm j\ .660$ $-.00705 \pm j\ .823$ $-20.$ $-40.$	.0722	9.75	9.28	-	-
q, θ, α	0.107 0.0997 0.228 -193 -53.6 -67.9	$-20.9 \pm j\ 43.3$ -18.9 -2.07 $-0.0607 \pm j\ 0.0537$ -0.2784	.0190	3.40	3.43	63.8	54.5
$q, \theta, \alpha-\alpha_g$	-1.82 -0.188 0.0262 -13.1 -41.3 -4.02	-36.6 $-11.7 \pm j\ 10.8$ -0.223 -2.61 -0.050 -0.2784	.0150	1.98	1.92	7.78	5.84
$q, \theta, \alpha, \alpha_g$	-0.310 -0.191 0.0442 -0.219 -1.02 -52.5 -416 3.14	$-20.2 \pm j\ 63.7$ -18.4 -3.91 -0.222 -0.0377 -0.2784	.000528	2.78	2.52	71.4	60.7
$q, \theta, \alpha-\alpha_g, \alpha_g$	-0.575 0.286 0.0175 -0.0427 -150 -83.3 -4.25 0.530	$-20.4 \pm j\ 30.4$ -20.9 -1.11 -0.139 -0.0995 -0.2784	.0143	2.83	2.78	46.7	39.5

Table 2

Effect of various controller configurations

Initial gains set equal to zero, sensor noise accounted for in design

Feedback Configuration	gain matrix	eigenvalues	J*	Vertical acceleration rms values ft/sec², No sensor noise: n_{z1}	No sensor noise: n_{z2}	w. sensor noise: n_{z1}	w. sensor noise: n_{z2}
Open loop	[0]	$-1.45 \pm j\ .660$ $-.0075 \pm j\ .0823$ $-20.$ $-40.$	.0722	9.75	9.28	-	-
q, θ, α	0.00721 -0.0299 -0.0343 -0.0239 -0.0204 -0.539	-39.85 -20.05 $-1.48 \pm j\ .888$ $-.0360 \pm j\ .0877$ -0.278	.0667	9.22	8.86	9.23	8.86
$q, \theta, \alpha - \alpha_g$	0.00411 -0.0368 0.0119 -0.0102 0.0175 -1.18	-39.7 -20.0 $-1.57 \pm j\ .846$ $-.0444 \pm j\ .0822$ -0.2784	.0478	4.43	4.25	4.42	4.25
$q, \theta, \alpha, \alpha_g$	0.00518 -0.0661 -0.186 -0.141 -0.0435 -0.0201 -0.140 1.18	-40.0 -20.1 $-1.40 \pm j\ 1.22$ $-.0575 \pm j\ .0913$ -0.2784	.0503	4.74	4.23	4.75	4.24
$q, \theta, \alpha - \alpha_g, \alpha_g$	.00468 -0.0398 -0.00984 -0.0451 -0.0100 -0.00753 -0.930 0.794	-39.8 -20.0 $-1.52 \pm j\ .886$ $-.0470 \pm j\ .0832$ $-.2784$	.0414	6.52	6.39	6.51	6.39

Table 3a Effects of Initial Gain Selection for J_2

$$y^T = (u,\ \theta,\ n_{zcg},\ q_{cg},\ \alpha_v,\ W_g)$$

Initial gains	J_2	Gains						eigenvalues	
OPEN LOOP	164	-1.69	-604	3.42	.0355	2.84	-.674	-2.19±j65.4 -10.8±j50.3 -1.78±j37.9 -2.93±j32.3 -8.73±j20.3	-24.2±j1.46 -.00595±j1.23 -41.8 -2.35 -.348
		.565	685	.568	1.48	3.71	1.15		
		-.0150	30	1.42	-.197	-.776	.236		
OPTIMAL GAINS FOR $n_{zcg}, q_{cg}, \alpha_v, W_g$	162	.105	-25.9	.870	-.235	1.71	-.345	-3.15±j63.4 -9.60±j49.6 -1.07±j36.1 -5.67±j32.2 -5.30±j23.8 -22.2±j12.6	-.00320±j.00989 -40.0 -12.9 -.316
		.106	38.5	-2.93	2.54	3.93	.932		
		-.791	-1.62	2.06	-1.30	.0656	-.00322		
OPTIMAL GAINS FOR $n_{zcg}, q_{cg}, \alpha_v, W_g, u, n_{zwt}$	144	.402	1.78	1.17	.235	11.2	-3.91	-2.64±j63.9 -10.1±j49.3 -3.36±j36.1 -.791±j34.9 -8.31±j22.6 -23.8±j8.21	-4.22±j2.91 -40.5 -.00152 -.1984
		.0517	18.9	-3.72	2.47	2.83	1.45		
		-.0208	54.8	.699	-1.26	-4.77	1.79		

Table 3b Effects of Initial Gain Selection for J_2

Covariances

Initial Gains	n_{z_p} (g^2)	n_{zcg} (g^2)	BMR $(n^2\text{-}m^2)$	SR $n^2\text{-}m^2$	δ_{SA} deg^2	δ_z deg^2	δ_e deg^2
OPEN LOOP	.1265E-03	.2019E-03	.7485E+10	.7171E+08	.4855E+00	.5344E+00	.4894E-02
OPTIMAL GAINS FOR n_{zcg}, q_{cg}, α_v, w_g	.1236E-03	.2022E-03	.7193E+10	.9440E+08	.4277E-01	.1603E+01	.3454E-02
GAINS FOR n_{zcg}, q_{cg}, α_z, w_g, u, n_{zwt}	.1126E-03	.1760E-03	.7060E+10	.9750E+08	.2398E-01	.1752E+01	.3454E-02

Table 4a Feedback Design Study for J_2, Cruise Condition

$Q = \mathrm{DIAG}\ (10^6, 10^6, 0) \qquad R = \mathrm{DIAG}\ (4., 6., 25.) \times 10^{-6}$

OUTPUT FEEDBACK CONFIGURATION	J	# Iter-ations	K such that $\begin{matrix}\delta_{SA}\\ \delta_z\\ \delta_e\end{matrix} = -Ky$	EIGENVALUES	
OPEN LOOP	7477		0	-4.90±j61.6 -11.4±j50.9 -1.76±j41.5 -3.02±j32.7 -5.89±j22.3	-1.52±j1.66 -0.00689±j.0465 -40. -30. -20.
n_{zcg}	1132	9	1.047 -7.698 4.889	-1.30±j64.4 -9.50±j50.4 -9.08±j35.6 -3.46±j34.1 -26.0±j5.98	-2.30±j23.4 -1.24±j3.95 -0.0063±j0.0235 -40.4
n_{zcg}, W_g	241	13	3.063 0.3072 -1.670 2.253 2.253 -0.00839	-1.71±j65.2 -10.7±j50.5 -2.86±j38.4 -3.92±j31.5 -7.01±j20.9	-1.41±j3.61 -0.0064±j0.0292 -41.0 -26.5 -22.9
n_{zcg}, W_g, q_{cg}	169	22	0.527 0.230 -0.306 -4.02 2.37 2.85 2.17 0.0229 -1.54	-30.1±j63.2 -9.35±j49.6 -0.978±j36.0 -7.00±j31.8 -22.5±j13.9	-3.90±j24.2 -0.00756±j0.0187 -39.8 -11.6 -2.53

Table 4a (Continued)

OUTPUT...	J	#Iter..	K such that....	Eigenvalues
$n_{zcg}, W_g, q_{cg}, \alpha_v$	153	17	.578 -4.12 0.0648 11.46 -3.04 3.04 5.41 -1.31 .00571 0.912 -0.967 -2.25	-3.46±j63.6 -7.44±j22.1 -10.5±j49.5 -2.82±j3.32 -2.66±j36.2 -.00853±j0.0561 -.00895±j34.2 -39.9 -26.9±j7.77
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT} x10^{-2}$	68.9	26	.771 -4.64 -.539 9.25 -51.9 -12.5 2.33 .435 3.72 94.2 -.405 1.47 -.894 -3.37 19.4	-.00845±j56.6 -.00760±j0.0748 -10.7±j49.4 -103 -6.07±j34.6 -23.6 -2.24±j32.6 -16.1 -8.04±j20.89 -2.51
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT} x10^{-2}, u$	67.7	9	.758 -4.67 -.634 9.24 -53.4 0.210 -12.5 2.31 .406 3.72 94.1 0.010 -.236 1.49 -.629 -3.37 19.9 0.100	-.00162±j56.6 -3.50±j2.64 -10.9±j49.6 -106 -6.17±j34.9 -22.5 -2.31±j32.4 -23.5 -7.87±j21.1 -0.0202 -0.00217
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT} x10^{-2}, \theta$	65.4	31	1.27 -4.74 -1.56 9.10 -53.5 1600 -13.5 2.41 1.93 3.75 100.0 -826 -.526 1.48 -.925 -3.39 17.6 -231	-.118±j56.8 -106 -10.7±j49.2 -26.2 -5.08±j33.8 -0.0000624 -10.8±j21.8 -0.0916 -6.91±j7.19 -6.68 -3.29±j32.9
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT} x10^{-2}$, BMR $x10^{-5}$	68.1	11	.790 -4.65 -.650 9.24 -53.0 .0111 -12.6 2.34 .507 3.69 94.2 .00573 -.294 1.47 -.562 -3.35 20.5 -.00758	-.000711±j56.7 -2.98±j3.07 -10.9±j49.6 -.00794±j0.0734 -6.09±j35.0 -106 -2.27±j32.4 -7.86±j21.2 -23.6±j1.62

Table 4b Feedback Design Study for J_2, Cruise Condition

$Q = \text{DIAG}\ (10^6,\ 10^6,\ 0)$ $R = \text{DIAG}\ (4.,\ 6.,\ 25.) \times 10^{-6}$

FEEDBACK CONFIGURATION	$n_{zp}(g^2)$ COVARIANCE	$n_{zcg}(g^2)$ COVARIANCE	$BMR(n\text{-}m)^2$ COVARIANCE	$S_R(n\text{-}m)^2$ COVARIANCE	$\delta_{SA}(dg)^2$ COVARIANCE	$\delta_z(dg)^2$ COVARIANCE	$\delta_e(dg)^2$ COVARIANCE
OPEN LOOP	0.7430E-02	0.7525E-02	0.1128E+11	0.1121E+09	0	0	0
n_{zcg}	0.8371E-03	0.1427E-02	0.4442E+10	0.4128E+08	0.1808E-02	0.9112E-01	0.3846E-01
n_{zcg}, W_g	0.2072E-03	0.2750E-03	0.7433E+10	0.9555E+08	0.5581E-01	0.1504E+01	0.2522E-02
n_{zcg}, W_g, q_{cg}	0.1239E-03	0.2146E-03	0.7003E+10	0.9455E+08	0.1915E-01	0.1579E+01	0.3509E-02
$n_{zcg}, W_g, q_{cg}, \alpha_v$	0.1402E-03	0.1649E-03	0.6696E+10	0.9870E+08	0.4223E-02	0.1911E+01	0.3829E-02
$n_{zcg}, W_g, q_{cg}, \alpha_v$, $n_{zwT} \times 10^{-2}$	0.6874E-04	0.6902E-04	0.4774E+10	0.1088E+09	0.4989E+00	0.3356E+01	0.1691E-01
$n_{zcg}, W_g, q_{cg}, \alpha_v$, $n_{zwT} \times 10^{-2}, u$	0.6616E-04	0.6917E-04	0.5220E+10	0.1106E+09	0.8124E+00	0.3422E+01	0.2257E-01
$n_{zcg}, W_g, q_{cg}, \alpha_v$, $n_{zwT} \times 10^{-2}$, θ	0.6310E-04	0.6773E-04	0.5195E+10	0.1084E+09	0.2625E+01	0.4477E+01	0.3379E-01
$n_{zcg}, W_g, q_{cg}, \alpha_v$, $n_{zwT} \times 10^{-2}$, $BMR \times 10^{-5}$	0.6667E-04	0.6959E-04	0.4882E+10	0.1098E+09	0.4650E+00	0.3413E+01	0.1652E-01

Table 5a Feedback Design Study for J_1, Cruise Condition

$Q = \text{DIAG}\ (0.,0.,10^6) \qquad R = \text{DIAG}\ (4.,6.,25.) \times 10^{-6}$

OUTPUT FEEDBACK CONFIGURATION	J	K such that $\begin{matrix}\delta_{SA}\\ \delta_z\\ \delta_e\end{matrix} = -Ky$	EIGENVALUES	
OPEN LOOP	.564E+16	0	-4.9±j61.6 -11.4±j50.9 -17.6±j41.5 -30.2±j32.7 -5.89±j22.3	-1.53±j1.66 -.00689±j0.0465 -40 -20 -30
n_{zcg}	.202E+16	-5.99 -6.04 5.92	-11.3±j57.2 -1.10±j49.2 -16.1±j45.9 -1.82±j33.4 -.595±j24.6	-24.2±j5.19 -1.20±j3.83 -.00629±j.0230 -38.4
n_{zcg}, W_g	.214E+15	-1.44 -6.37 -1.44 5.56 -1.02 .755	-5.86±j59.6 -11.7±j51.3 -1.33±j43.2 -2.66±j33.2 -4.99±j23.1	-.00245±j.0883 -39.3 -31.2 -21.1 -2.48 -0.48
n_{zcg}, W_g, q_{cg}	.109E+15	1.56 -1.37 -3.29 -7.92 0.307 0.538 -3.90 0.549 -.434	-1.16±j64.5 -11.8±j49.4 -.0347±j42.8 -5.21±j32.0 -5.45±j25.6	-.00664±j0.759 -41.3 -29.8 -19.6 -7.81 -.1785
$n_{zcg}, W_g, q_{cg}, \alpha_v$	.664E+14	-1.64 -5.69 -1.69 23.3 -5.88 2.16 -2.41 -33.0 -3.24 -.515 -.960 -5.31	-4.43±j61.4 -11.4±j50.8 -0.720±j42.9 -.728±j31.8 -38.7 -25.9 -8.81±j25.6	-.00825±j.0754 -15.6 -9.94 -5.58

Table 5a (Continued)

OUTPUT FEEDBACK CONFIGURATION	J	K such that $\begin{matrix}\delta_{SA}\\ \delta_z\\ \delta_e\end{matrix} = -Ky$	EIGENVALUES	
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT} x10^{-2}$	.630E+14	-1.07 -5.56 -1.72 25.1 -7.82 -2.69 2.16 -1.36 -33.0 35.8 -2.87 .488 -.967 -6.08 5.63	$-9.31 \pm j59.3$ $-10.7 \pm j49.8$ $-.0142 \pm j43.8$ $-.813 \pm j32.0$ $-8.11 \pm j24.6$ $-12.0 \pm j5.16$	$-.00843 \pm j.0746$ -48.7 -15.5 -8.79
$n_{zcg}, W_g, q_{cg}, \alpha_v, u$	.649E+14	-1.66 -5.65 -1.49 23.4 .00986 -4.58 2.16 -2.07 -32.2 -.00108 -3.46 .486 -1.00 -5.80 -.0000129	$-4.98 \pm j61.2$ $-11.48 \pm j50.9$ $-.0455 \pm j42.8$ $-.617 \pm j31.8$ $-9.34 \pm j25.3$ $-11.9 \pm j1.35$	$-.00804 \pm j.0787$ -38.8 -26.4 -6.05
$n_{zcg}, W_g, q_{cg}, \alpha_v, \theta$	.661E+14	-1.76 -5.71 -1.61 24.7 -3.19 -4.83 2.21 -1.76 -35.4 1.59 -3.51 .461 -1.05 -6.58 -.660	$-5.01 \pm j61.4$ $-11.5 \pm j50.8$ $-.000188 \pm j42.6$ $-.576 \pm j31.8$ $-9.44 \pm j25.4$ $-10.9 \pm j4.93$	$-0.0138 \pm j0.0778$ -38.8 -27.3 -7.13
$n_{zcg}, W_g, q_{cg}, \alpha_v$, BMR x 10^{-5}	0.600E+14	-1.61 -5.55 -1.44 22.8 .419 -4.23 2.12 -2.04 -32.9 .0479 -3.31 .506 -1.01 -5.52 1.01	$-5.31 \pm j60.7$ $-10.8 \pm j51.2$ $-.000466 \pm j42.7$ $-36.6 \pm j2.94$ $-.671 \pm j32.0$ $-10.2 \pm j28.3$	$-.00517 \pm j0.0761$ -14.5 -8.84 -.642

Table 5b Feedback Design Study for J_1, Cruise Condition

$Q = \text{DIAG}\ (0.,\ 0.,\ 10^6)$ $R = \text{DIAG}\ (4.,\ 6.,\ 25.) \times 10^{-6}$

FEEDBACK CONFIGURATION	$n_{zp}(g)^2$ COVARIANCE	$n_{zcg}(g)^2$ COVARIANCE	$BMR(nm)^2$ COVARIANCE	$S_R(nm)^2$ COVARIANCE	$\delta_{SA}(dg)^2$ COVARIANCE	$\delta_z(dg)^2$ COVARIANCE	$\delta_e(dg)^2$ COVARIANCE
OPEN LOOP	0.7430E-02	0.7525E-02	0.1128E+11	0.1121E+09	0	0	0
n_{zcg}	0.3732E-02	0.1582E-02	0.4039E+10	0.4331E+08	0.5812E-01	0.5170E-01	0.5389E-01
n_{zcg}, W_g	0.1848E-01	0.8678E-02	0.4273E+09	0.1210E+09	0.1291E+02	0.9548E+01	0.1734E+00
n_{zcg}, W_g, q_{cg}	0.3757E-01	0.2232E-01	0.2173E+09	0.2854E+08	0.3537E+01	0.1010E+01	0.2495E+00
$n_{zcg}, W_g, q_{cg}, \alpha_v$	0.2924E-01	0.1223E-01	0.1328E+09	0.1200E+09	0.1649E+02	0.7843E+01	0.2331E+00
$n_{zcg}, W_g, q_{cg}, \alpha_v, n_{zwT}$	0.7553E+00	0.4391E+00	0.1259E+09	0.3062E+09	0.1727E+02	0.1306E+02	0.2938E+01
$n_{zcg}, W_g, q_{cg}, \alpha_v, u$	0.1968E+00	0.8606E-01	0.1297E+09	0.1576E+09	0.1708E+02	0.8976E+01	0.7916E+00
$n_{zcg}, W_g, q_{cg}, v, \theta$	0.4138E+02	0.1730E+02	0.1282E+09	0.9244E+10	0.1233E+03	0.2846E+03	0.1401E+03
$n_{zcg}, W_g, q_{cg}, \alpha_v, BMR \times 10^{-5}$	0.1866E+02	0.8563E+01	0.1199E+09	0.4008E+10	.5860E+02	.1180E+03	.5999E+02

Table 6a Full State Feedback (Cruise Condition)

Q	R	J	EIGENVALUES	
$\begin{bmatrix}10^6 & & \\ & 10^6 & \\ & & 0\end{bmatrix}$	$\begin{bmatrix}.04 & & \\ & .06 & \\ & & .25\end{bmatrix}$	2.17	-112 ±j34.3 -12.8±j57.4 -2.96±j40.2 -36.4±j5.44 -20.7±j13.8	-.253±j.225 -4775 -2342 -27 -.0109 -.000837
$\begin{bmatrix}10^6 & & \\ & 10^6 & \\ & & 0\end{bmatrix}$	$\begin{bmatrix}4 & & \\ & 6 & \\ & & 25\end{bmatrix}$	33.6	-154 ±j75.2 -12.8±j57.3 -2.96±j40.2 -36.5±j5.51 -20.7±j13.8	-.753±j.740 -474 -110 -27 -.0108 -.00145

Table 6b Full State Feedback (Cruise Condition)

Q	R	$n_{zp}(g)^2$ COVARIANCE	$n_{zcg}(g)^2$ COVARIANCE	$BMR(n\text{-}m)^2$ COVARIANCE	$S_R(n\text{-}m)^2$ COVARIANCE	$\delta_{SA}(dg)^2$ COVARIANCE	$\delta_z(dg)^2$ COVARIANCE	$\delta_e(dg)^2$ COVARIANCE
$\begin{pmatrix} 10^6 & & \\ & 10^6 & \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} .04 & & \\ & .06 & \\ & & .25 \end{pmatrix}$	.1821E-06	.3781E-06	.9973E+10	.1297E+09	.1483E+02	.1381E+02	.3727E+00
$\begin{pmatrix} 10^6 & & \\ & 10^6 & \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & & \\ & 6 & \\ & & 25 \end{pmatrix}$	.2972E-04	.1588E-04	.7078E+10	.8506E+08	.9280E+00	.1778E+01	.2201E-01

SENSOR NOISE STUDY
CRUISE CONDITION $J = J_2$
$y = (n_{zcg}, q_{cg}, \alpha_v, W_g)^T$

Table 7a

SENSOR NOISE STANDARD DEVIATIONS
$\sigma\, q_{cg}$: $.5^{o/s}$ $\sigma\, \alpha_v$: $.2^o$

DESIGN TYPE	J_2	K SUCH THAT $\begin{matrix}\delta_{SA}\\ \delta_z\\ \delta_e\end{matrix} = -Ky$				EIGENVALUES	
NO SENSOR NOISE IN OPTIMIZATION PROCEDURE	168	.966	-.163	1.31	-.266	-2.64±j63.7	-40.2
						-9.37±j49.6	-2.26
		-4.09	2.28	-.854	2.68	-1.10±j36.0	-.0156
						-6.83±j31.2	-13.0
		2.32	-1.53	.0656	-.00130	-4.24±j24.3	-.0000631
						-21.7±j14.0	
SENSOR NOISE ACCOUNTED FOR IN THE OPTIMIZATION PROCEDURE	241	3.07	0.0000461	-0.0000462	0.311	-1.71±j65.2	-0.00644±j0.0292
						-10.7±j50.5	-41.2
		-1.67	-0.0000168	0.000420	2.25	-2.85±j38.4	-26.5
						-3.92±j31.6	-22.9
		2.26	-0.000273	0.000236	-0.00904	-7.01±j21.0	
						-1.41±j3.61	

Table 7b

COVARIANCE / Design	n_{zp} (g^2)	n_{zcg} (g^2)	BMR $(n\text{-}m)^2$	S_R $(n\text{-}m)^2$	δ_{SA} $(dg)^2$	δ_z $(dg)^2$	δ_e $(dg)^2$
DESIGNED WITHOUT NOISE TESTED WITHOUT NOISE	.1223E-03	.2131E-03	.8406E+10	.1150E+09	.1008E+00	.1625E+01	.3919E-02
DESIGNED WITHOUT NOISE TESTED WITH NOISE	.3226E+01	.6966E+00	.1127E+13	.1747E+11	.6492E+02	.4869E+02	.1414E+02
DESIGNED WITH NOISE TESTED WITHOUT NOISE	.2069E-03	.2752E-03	.7439E+10	.9551E+08	.3431E-01	.5674E-01	.1499E+01
DESIGNED WITH NOISE TESTED WITH NOISE	.2070E-03	.2752E-03	.7439E+10	.9551E+08	.5674E-01	.1499E+01	.2515E-02

Table 8 C^* response design

$$y = (n_{zcg}, q_{cg}, \alpha_v)^T$$

$$u = -Ky$$

Q	Gains			Eigenvalue
10^6	-.4321	1.087	16.00	-3.47±j63.6
	-6.484	-5.453	-29.14	-17.6±j43.6
	5.500	-2.703	-45.73	-2.00±j37.8
				-.000337±j26.4
				-5.86±j16.6
				-.00749±j.0933
				-40.9
				-31.3
				-10.5
10^8	-.2254	1.078	15.70	-3.07±j63.8
	-7.006	-5.491	-28.39	-17.6±j43.4
	5.461	-2.724	-45.38	-3.74±j46.5
				-.00210±j26.4
				-5.97±j16.5
				-.00752±j.0929
				-41.0
				-31.3
				-10.5

Table 9a SENSITIVITY STUDY

$$Q = \begin{pmatrix} 10^6 & & \\ & 10^6 & \\ & & 0 \end{pmatrix}; \; \underline{y} = (n_{zcg}, q_{cg}, \alpha_v, W_g)^T \qquad K = \begin{bmatrix} .966 & -.163 & 1.31 & -.266 \\ -4.09 & 2.28 & -.854 & 2.68 \\ 2.32 & -1.53 & .0656 & -.0013 \end{bmatrix}$$

CRUISE GAINS EVALUATED AT VARIOUS FLIGHT CONDITIONS

CONDITION	COV n_{zp}	COV n_{zcg}	EIGENVALUES	
CLIMB	.3733E-03	.3183E-03	$-2.58\pm j62.0$ $-5.40\pm j46.8$ $-2.81\pm j39.6$ $-2.08\pm j32.0$ $-3.18\pm j21.7$ $-23.7\pm j3.36$	$-.0122\pm j.0271$ -40.0 -6.68 -1.08
CRUISE	.1223E-03	.2131E-03	$-2.64\pm j63.7$ $-9.37\pm j49.6$ $-1.10\pm j36.0$ $-6.83\pm j31.2$ $-4.24\pm j24.3$ $-21.7\pm j14.0$	-40.2 -2.26 $-.0156$ -13.0 $-.0000631$
LAND	.2012E-02	.1145E-02	$-2.64\pm j61.7$ $-5.14\pm j46.4$ $-2.75\pm j40.3$ $-1.89\pm j32.0$ $-3.09\pm j21.6$ $-.0127\pm j.0635$	-40.0 -24.4 -24.1 -4.30 $-.983$

Table 9b SENSITIVITY STUDY DIAG(.03355,1.957,.00154,0,0,74740,0,8880,0,20440,0,6878,0,8678,0,0,0) = E(WWT)

$$Q = \begin{bmatrix} 10^6 & & \\ & 10^6 & \\ & & 0 \end{bmatrix} \qquad K = \begin{bmatrix} -.519 & -.834 & -8.33 & 3.50 \\ -1.24 & .706 & 1.48 & 1.94 \\ 2.94 & -.804 & -6.97 & 2.62 \end{bmatrix}$$

CRUISE GAINS WITH PROCESS NOISE

FLIGHT CONDITION	COVARIANCE n_{zp}	COVARIANCE n_{zcg}	EIGENVALUES	
CLIMB	0.5787E-02	0.5748E-02	-3.42±j61.0 -5.11±j46.6 -3.50±j40.8 -3.56±j33.9 -2.48±j26.4 -.818±j5.58	-.000723±j.151 -39.6 -28.7 -21.1
CRUISE	0.2124E-03	0.3388E-03	-4.94±j61.5 -9.40±j50.0 -2.37±j38.4 -8.80±j35.3 -3.87±j22.7 -23.1±j5.16 -1.68±j10.5	-.00747±j.0704 -38.7
LAND	0.9490E-02	0.8529E-02	-3.26±j60.9 -4.85±j46.0 -3.37±j41.4 -3.27±j33.7 -2.37±j21.4 -.763±j4.63	-.00117±j.179 -39.7 -29.6 -20.3